2015-12-01

Mathematical Interactions between Teachers and Students in the Finnish Mathematics Classroom

Paula Jeffery Prestwich
Brigham Young University - Provo

Mathematical Interactions Between Teachers and

Students in the Finnish Mathematics Classroom

Paula Jeffery Prestwich

A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of

Master of Arts

Douglas L. Corey, Chair
Steven R. Williams
Blake E. Peterson

Department of Mathematics Education

Brigham Young University

December 2015

ABSTRACT

Mathematical Interactions Between Teachers and
Students in the Finnish Mathematics Classroom

Paula Jeffery Prestwich
Department of Mathematics Education, BYU
Master of Arts

The Finnish school system has figured prominently in the PISA international assessments for over 10 years. Many reasons are given for Finnish success yet few of them focus on what is happening in the mathematics classroom. This study addresses the question of "What does mathematics instruction in the Finnish mathematics classroom look like?" Eight Finnish mathematics classes, from 6th – 9th grade were recorded, translated, and analyzed using the Mathematical Quality of Instruction (MQI) 2013 video coding protocol. Other aspects and observations of these classes also are discussed. Although the study is small, this study gives a view into the nature of some Finnish mathematics classrooms.

Keywords: Finland, mathematics instruction, interaction, student, teacher, MQI

ACKNOWLEDGEMENTS

Special thanks to the teachers, students and schools, who allowed me to come video and observe their classrooms. We were grateful for the warm welcome that we received. I was impressed with their dedication and efforts to educate the children of Finland. I wish them the best in all their educational endeavors.

Special acknowledgement should be given Markku Hannula and Enrique Garcia of the University of Helsinki. I am grateful for the use of video from their projects to aid in my research. I am also grateful for their help and feedback given over the course of my research.

I also need to acknowledge my Finnish mother, Raili Jeffery, who spent many hours with me reviewing and translating letters, e-mails, and video. She made this thesis possible. She also helped me as I collected video in Finland, which was a good experience for both of us.

I also need to express my gratitude for the support of my Finnish relatives who fed us, housed us, and helped with transportation while in Finland, especially my uncle Reino Leppäniemi and my aunt, Riitta Lindell. Kiitoksia!

Thank you to my advisor, Doug Corey, for his support and help throughout this research. Thank you for your patience. And thank you to all of the BYU Mathematics Education faculty, who helped me become better. I wish to also thank Steve Williams, whose wise and compassionate counsel helped me to persevere and not give up.

And lastly, I am grateful to my husband and children, who are the reason I keep going and who are glad I'm done. It's been a tough road.

Table of Contents

List of Figures

List of Tables

Chapter 1: Rationale

Several international studies and assessments, like TIMSS 1999 and PISA (OECD, 2003, 2006), have drawn attention to the fact that, while the US school system overall is average, other school systems of the international community seem to be experiencing high levels of success in the education of their children, as measured by scores on these studies and assessments (Stigler & Hiebert, 1999; Schliecher, 2007). Researchers have explored the school systems of other nations, like Japan, China, and Singapore, whose students have performed well on international assessments in an effort to understand the reasons for their success (Corey, Peterson, Lewis, & Bukarau, 2010; Fan & Zhu, 2007; Jacobs & Morita, 2002; Ma, 1999; Stevenson & Stigler, 1992; Stigler & Hiebert, 1999). The hope is that as we come to understand these other school systems we might be able to find ways to improve education in the U.S.

Another school system that has figured prominently in the PISA tests of the last 10 years is the Finnish school system. On the 2003 PISA exam, which focused on mathematics, Finland placed 1st of the 28 OECD (Organization for Economic Co-operation and Development) countries, which include such countries as the United Kingdom, U.S., Germany, and Japan. They have also placed 1st on the other PISA tests that focus on science and reading. While there has been much interest in the Finnish school system from the media, government officials of other countries, and administrators of non-Finnish school systems, outside research focusing on what is happening in Finnish mathematics classrooms seems to be lacking.

Finnish researchers and educators have offered a number of reasons for the success of Finnish students on PISA. Pasi Sahlberg (2011) of the Finnish ministry of education places great emphasis on the level of equity that Finland has achieved as they have reformed their school system over the last 40 years. This development of equity, as revealed through the PISA assessments, has also caught the eye of the international community (OECD, 2003, 2006; Sahlberg, 2011; Schleicher, 2007).

At the end of the 1960's the Finnish school system was highly fractured with very unequal learning opportunities for their children. Education is considered in Finland as a basic human right guaranteed in the Finnish constitution, which was enacted in 1919. So, in order to improve the learning opportunities, and thus the future employment opportunities, for Finnish children, Finland made educational change a priority. Finland changed their system of education by mostly eliminating private schools, providing better school opportunities for students in rural areas, providing increased access to higher education for all students, increasing the required educational level of teachers, and creating national curricular goals to help guide the creation of curriculum at the local level. Through persistence, described by the Finnish word *sisu*, Finland has not only changed their school system to achieve greater equity of learning opportunities for all their children, but has become one of the best school systems in the world in science and mathematics, according to PISA (Reinikainen, 2012; Sahlberg, 2011).

Additional reasons discussed in the literature for Finnish success in mathematics include the use of special education in Finnish schools, respect for teachers, and high reading skills. All students have access to additional help, if needed, through special education, regardless of disability or lack thereof (Kupiainen, Hautamäki, Karjalainen, 2009; Sahlberg, 2011). The respect teachers are given in Finland is another reason often cited for Finnish success (Sahlberg, 2011). High literacy and reading skills also might have a factor in a country with essentially zero illiteracy among native adults (Kupiainen, Hautämaki, Karjalainen, 2009). The reading ability of the students could make understanding mathematics and mathematical problems easier. Other possible reasons for Finnish success are the independence given teachers similar to that found in

U.S. charter schools, the socioeconomic homogeneity, lack of high-stakes testing, textbook materials, and the emphasis on problem solving, etc. (Kupiainen, Hautämaki, Karjalainen, 2009; Sahlberg, 2011, Lampiselkä, Ahtee, Pehkonen, Meri, Eloranta, 2007). Some of the literature discusses the use of problem solving in the Finnish mathematics classroom, yet doesn't give a clear picture of how problem solving is implemented (Lampiselkä, Ahtee, Pehkonen, Meri, Eloranta, 2007; Pehkonen & Rossi, 2007).

Many reasons are given for Finnish success in mathematics yet few of them focus on what is happening in the mathematics classroom. Cohen, Raudenbush, and Ball (2003) state "Instruction consists of interactions among teachers and students around content, in environments." The explanations given for Finnish success lack much detail about the interactions between teacher and students around the content and descriptions of the environment in the classroom. While factors, inside and outside of the classroom, feed into the instruction, the interactions between teacher and student are at the center of classroom instruction and are the major factor in teaching quality and thus student achievement gains (Ball and Hill, 2008; Hiebert, 2013). Reinikainen (2012) also comments that the statistics would seem to indicate that the instructional quality in the Finnish classroom is an important factor in student learning outcomes. This suggests an important question: What does mathematics instruction in the Finnish mathematics classroom look like? In this study, I investigate the mathematical instruction of Finnish classrooms. I also discuss other aspects of instruction in the Finnish mathematics classroom that might impact the instruction.

Chapter 2: Literature Review

Finland has made many changes to their school system in the last century that have not only helped change their school system significantly but also have significantly changed the quality of their lives. Finland has gone from having over three-fourths of the adult population with no more than six years of basic education to almost three-fourths of the adult population having at least a basic education, which was increased to 9 years of schooling from 6 years (Sahlberg, 2011). The people of Finland have made the transition from a poor agrarian economy to a more modern, technology-based economy which has been aided by the improved education of their children and adults. The history of the Finnish school system gives a backdrop that helps better understand not only Finnish education but the motivations of the Finnish people with respect to education. In this section I will discuss the history of the Finnish school system, changes made to the school system, features of the current Finnish school system, and what we know about Finnish mathematics instruction.

The Finnish School System – After World War II

Post-WW II Finland experienced many troubles and difficulties. With significant losses to the population during WWII, loss of land and resources to the Soviet Union, and the need to change from an agrarian to an industrial society, Finland saw many political and social changes. They recognized, as well, the need to reform education if they were to build a better society. Realistically, educational opportunities were available at that time mainly to students in towns and municipalities and were often limited to the six or seven years of basic education (or less, in more rural areas). Some students, in larger towns and municipalities, also had access to continuing education through vocational school or university. Schools in Finland were a combination of state-run institutions and privately run schools. During the 1950's the Finnish government began granting state subsidies to private institutions so that more educational opportunities would be available to the people of Finland, which also allowed the government to have more influence over schools in the private sector (Niemi, 2012; Sahlberg, 2011).

At the end of the 1960's the Finnish school system was still very fractured with unequal educational opportunities, but with more schools, both public and private (Sahlberg, 2011). Although learning opportunities for children were more widely available, the amount and quality of education was still unequal. Ideally, students had two possibilities for education beyond the years of basic education but these opportunities were not available to all. Students could pursue vocational training in preparation to enter the work force or pursue further schooling followed by study at a university. In 1970, 79% of the adult population of Finland only had a basic education (Sahlberg, 2011). Hard economic times caused policy makers to recognize that, if they were going to improve industry and technology and improve the quality of life for the populace as a whole, they still needed a better educated work force. A redesign of the education system was begun with the goal of more equitable learning outcomes for all of Finland's children.

The Finnish School System – Now

Finland is part of the European Union with approximately 5 ½ million citizens (www.stat.fi, 2012). Finland has three native languages: Finnish, Swedish and Sami. The percentage of the population of native Finnish speakers is 90% (www.stat.fi, 2012). Approximately half of the remaining 10% are native Swedish speakers. Sami speakers make up a small percentage of the remaining population which also includes other foreign ethnic populations, for example, Russian, Estonian, and Somali.

Finland's system of education offers learning opportunities for all. Basic, or compulsory, education, called peruskoulu, is for nine years and begins at the age of 7, with an optional preschool year for 6 year olds, which 98% of children are enrolled in. After nine years of basic education students have three choices: to begin vocational education, attend a 10th year of basic education to prepare for university, or to begin a university education. Students can also switch between vocational and university training at a later point, if desired, which was not possible in the older educational system. The education system also has increased in adult educational opportunities as well.

According to PISA 2009, Finland has achieved a great deal of equity in their learning outcomes. Education, beyond basic education, is found throughout Finland, not just in population centers. In 2010, 67% of Finnish adults had more than a basic education, which now includes nine years instead of six, an increase of 46% of the adult population compared to 1970 (Sahlberg, 2011). PISA 2009 results also showed Finland to have the lowest difference between low and top performing students, as well as no significant difference between males and females in math, though there was a difference in reading (Sahlberg, 2011). In 2009, the lowest achievers in Finland on PISA scored 70 points higher than the OECD average (Kupari, Reinikainen, & Törnroos, 2007; Reinikainen, 2012).

Pasi Sahlberg (2011) places great emphasis on the level of equity that Finland has achieved as they have reformed over the last 40 years. One of the contributing factors in Finland's success in achieving equitable learning outcomes is the availability of extra help for students through the use of special education. Special education is a resource for all students, not just for students who qualify. Any student who is struggling can receive additional help and support beginning in first grade and continuing throughout peruskoulu, basic education. Fifty percent of Finnish students receive some form of additional help by the time they reach the 8th grade (Sahlberg, 2011).

As part of educational reform, a master's degree is required for teachers of basic and upper secondary education. Competition is high for entry into teaching programs. Only about 800 of the about 6500 applicants to the teaching program are accepted every year (Toom &

Husu, 2012). The typical length of time to earn a master's degree in education is five years and includes both pedagogy and subject courses. The education master's program is a 5 year program with a three year bachelor's degree program and a two year master's degree program (Niemi, 2012). The program consists of academic, research, and pedagogical studies, as well as other studies that include language and communication studies. As part of their pedagogical studies, students have multiple opportunities to teach in the classroom and interact with students with different backgrounds and psychological orientations. Teaching students are also encouraged to reflect on their experiences and teaching practice. The goal in teacher education, according to Tirri (2012) is to "educate autonomous, professional teachers who build their practice on research-based knowledge and ethical values" (p. 64).

Finnish schools and teachers seem to have a higher degree of independence in the education of the students within their classrooms as compared to the U.S. Although Finland has a national curriculum, each municipality and school designs their own curriculum with the national curriculum as a guide, but with consideration of the local population and their needs (Kumpulainen & Lankinen, 2012, Sahlberg, 2011). Teachers in Finland are not subject to evaluation, such as high-stakes testing like in the U.S. In fact standardized testing is rare in Finnish schools. When evaluation of the teacher occurs improvement of student learning is the objective (Kumpulainen & Lankinen, 2012).

Another aspect of the Finnish school system that has been suggested as one of the causes for Finnish student success is the teacher to student ratio (Reinikainen, 2012; Sahlberg, 2011). The ratio of students to teaching staff in primary education is 11.4 students per teaching staff member. In lower secondary education the ratio is 10.6 per teaching staff member (Reinikainen, 2012).

Two aspects of the Finnish school system are puzzling when considering Finnish performance in international assessments (PISA 2006; Reinikainen, 2012; Sahlberg, 2011). The first aspect is that Finland spends less on education than many of the other high-achieving school systems. The second aspect is that instructional time per year in Finland is 5752 hours, which is significantly less than the OECD average of 6777 hours (Reinikainen, 2012). This is part of what Pasi Sahlberg calls the "Finnish Paradox" and states that "Less is More." He argues that because there is less instructional time in the classroom that teachers have more time to plan and prepare for their classes and reflect on their teaching. These statistics would seem to indicate that the instructional quality in the classroom is the important factor in the outcomes of students on international assessments and that studying the instruction and the classroom interactions would help us to understand better their success (Reinikainen, 2012).

Finnish Mathematics Instruction

Not much literature is available about Finnish mathematics classroom instruction. From the literature available, instruction, at the secondary level, appears to be similar to the United States in that instruction is more traditional in nature with a review of homework at the beginning of class, followed by instruction on a new concept to be learned (Maijala, 2006; Pehkonen & Rossi, 2007). After the instruction, teachers give problems to be solved, often from the textbook, and then students are assigned homework. This traditional pattern has begun to change, especially for teachers of math in the higher grades, with the introduction of theories, like constructivism (Pehkonen and Rossi, 2007). The introduction of these ideas has led to an increasing awareness by teachers of student understanding, thoughts, and interactions (Pehkonen and Rossi, 2007; Tirri, 2012).

New curricula ideas generally followed U.S. changes with a ten year time lag (Kupari, 1999; Lampiselkä, Ahtee, Pehkonen, Meri, Eloranta, 2007; Pehkonen, Hannula, Björkqvist, 2007). Because of this, problem solving was emphasized in Finland in the 1990's, about 10 years after problem solving began to be an educational goal in the United States. For at least 20 years problem solving has been a major goal in Finnish teacher education and education (Pehkonen, Hannula, Björkqvist, 2007). Teachers received training in problem solving through in-service meetings, seminars, teacher's guides, teaching journals, and pre-service training (Pehkonen, Hannula, Björkqvist, 2007). Textbooks, which are heavily used, also increased the amount of problem solving included by the end of the 1990's (Pehkonen, Hannula, Björkqvist, 2007). One such textbook that emphasized problem solving is *Mieti ja Laske*, translated "Think and Calculate".

Pehkonen, Hannula, Björkqvist (2007) distinguish between tasks and problems in problem solving, as used in Finland. Problems are a subset of tasks. Tasks are any mathematical or reasoning exercise given to students. What makes a task a problem is dependent on the background and situation of the person solving it. If a student has had some exposure to the problem before the problem might become a task since less effort would be required by the student to understand and find a solution. Pehkonen, Hannula, and Björkqvist (2007) explain that various forms of problems are used in Finland, including the Japanese style of open-ended questions. However, Pehkonen, Hannula, and Björkvist (2007) talk about how the teacher's attitudes are favorable towards problem solving but the time and effort required to implement problem solving in the classroom is a major hurdle.

Dr. Rebecca L. Seaberg of Bethel University spent a year visiting classrooms in Sweden and Finland (Seaburg, 2015). She wrote about a number of aspects of Finnish instruction already noted, for example, “Less is More,” or the “Finnish Paradox” (Sahlberg, 2011), the educational level of the teachers, respect for teachers, the more relaxed atmosphere in class, the curriculum, etc. Dr. Seaberg also observed interactive whiteboards in the classrooms though the teachers didn’t feel they had time to learn to implement them well. She also noted that Finnish mathematics classrooms were similar to U.S. traditional classrooms (Seaburg, 2015).

The literature on Finnish mathematics classrooms and instruction is limited. I have included information that I have found that I feel might help to better understand the context in which Finnish mathematics classrooms and instruction are set.

Chapter 3: Theoretical Framework

In this section I will discuss my views on teacher and instructional quality, the factors that affect teacher and instructional quality and how this is related to the interactions in the classroom. I will also explain the aspects of the mathematics and of the teacher and student(s) interactions that I will focus on in the Finnish mathematics classroom and why I feel those aspects are important.

Assessing teachers and teacher quality is a major topic among teachers, administrators, teacher educators, politicians, and even parents in the United States (Ball & Hill, 2008). Different approaches have been taken to assess teacher quality, most of which can be seen as distant from classroom practice (Ball & Hill, 2008; Hiebert & Grouws, 2007). One approach has been to assess the educational level of the teacher as a measure of teacher quality. Research has shown that educational level, at least in the U.S., only has limited reliability as an indicator when compared to student achievement and that mostly at the secondary level (Ball & Hill, 2008; Goldhaber & Brewer, 1997). A teacher’s knowledge has closer correlation to student achievement gains than the level of education attainment (Hill, Blunk, Charalambous, Lewis, Phelps Sleep & Ball, 2008). Other factors that have been used to assess teacher quality in the United States are teacher certification, years of experience, cultural responsiveness, and pedagogical skill. However, Ball and Hill (2008) argue that to assess teacher quality we must assess *teaching* quality. While factors, inside and outside the classroom, feed into the instruction, the interactions between teacher and student around the content are the major factor in teaching quality (Ball & Hill, 2008; Hiebert, 2013). It is quality instruction that promotes "opportunity to learn" which is at the core of student achievement gains (Ball & Hill, 2008; Hiebert & Grouws, 2007; National Research Council, 2001).

At the core of classroom instruction are the interactions between the teacher, student(s) and content (Cohen & Ball, 1999; Cohen, Raudenbush, Ball, 2003; Hiebert, 2013; Hiebert & Grouws, 2007; National Research Council, 2001). I feel that the interactions between the teacher and students need to be assessed to better understand teacher and instructional quality. The Instructional Triangle (National Research Council, 2001, see figure 1) is an illustration of these interactions. The depiction in figure 1 shows that students interact with each other, interact with the content and interact with the teacher. This idea is illustrated by Stigler and Hiebert (1999). In the *Teaching Gap*, Stigler & Hiebert (1999) share a mathematics professor's summary of the differences in interactions of German, Japanese, and U.S. teachers.

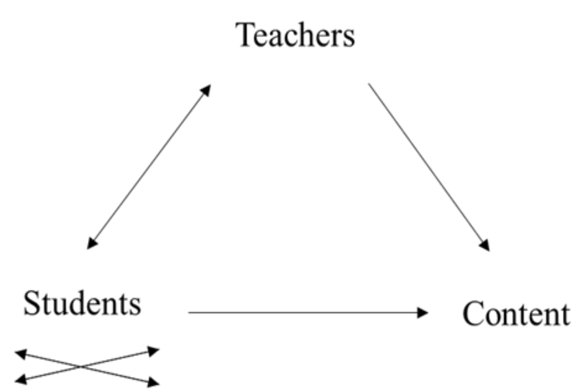

Figure 1. Instructional Triangle (National Research Council, 2001)

"In Japanese lessons, there is the mathematics on the one hand, and the students on the other. The students engage with the mathematics, and the teacher mediates the relationship between the two. In Germany, there is the mathematics as well, but the teacher owns the mathematics and parcels it out to students as he sees fit, giving facts and explanations at just the right time. In U.S. lessons, there are students and there is the teacher, I have trouble finding the mathematics; I just see interactions between students and teachers. " (p. 25, 26)

Although Stigler and Hiebert continue on and say that the content was not totally absent the level of difficulty of the problems was low with the mathematics being largely dependent on memorization and procedures (Stigler & Hiebert, 1999). However, this does illustrate the central importance of the mathematical content in the interactions that take place in the mathematics classroom.

My view is different than the instructional triangle (see figure 1) since I feel the central feature of the interactions (see figure 2) between the teacher and students and among students is the mathematical content. Teachers use their knowledge of the content, their pedagogical knowledge, and their knowledge of the students to craft the lesson and the interactions in the classroom (Ball, Thames & Phelps, 2008; Shulman, 1986). These interactions can take many forms both verbal and non-verbal. Some interactions around the content would seem to have a greater impact on

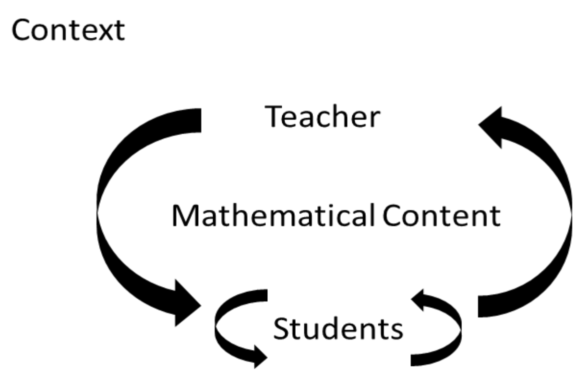

Figure 2. Teacher, Student and Content Interactions

student learning with the most important interactions having quality mathematics content at the center. Type and quality of the interactions around the content would need to be measured as well as the quality of the content of the lesson. Such interactions could take the form of direct instruction, remediation, individual and group work, whole class discussion and student contributions to lessons. Classroom interactions include verbal and non-verbal communication among the teacher and students around the content. These interactions are also affected by other factors, such as culture, classroom norms, classroom expectations, learning materials, societal expectations, socioeconomic pressures, and other mediating factors (Cobb, Wood, & Yackel, 1993; Hiebert & Grouws, 2007; Stigler & Hiebert, 1999). Although these factors are things to consider, my focus will be on the student and teacher interactions around the mathematical content.

Since the mathematical content is the central feature of interactions in the classroom, various factors of the mathematics and the interactions of the teacher and students around the content should be considered. Emphasis on deeper conceptual understanding of the mathematics is an important factor in the learning and understanding of mathematics by students (Hiebert & Grouws, 2007; Schoenfeld, 1992). To illustrate, I will share an example given by Heather Hill and other members of the Learning Mathematics for Teaching Project (2011). They give an example of a teacher who has her students explore the circumference of a circle through measuring the circumference of different circles. On the surface the lesson appears to follow ideas that research suggests are valuable to mathematical learning in the classroom, for instance, the teacher allows the students to explore the idea of circumference through measurement. However, much of the time is spent during the lesson on the students cutting out different size “pie” shapes and then measuring their circumference. The teacher then discusses what circumference is but closes the lesson with the idea that the circumference is “how far around you can go on a circle.” This lesson is lacking in respect to the depth of the mathematics covered and includes a lot of time in which students are not thinking deeply about the mathematics but are instead cutting out “pie” shapes. Amount and depth of the mathematics are important considerations when evaluating the interactions between teacher and students.

The mathematical language the teacher and students use is a major factor in considering the quality and depth of the mathematics and instruction. Language is the central medium through which instruction and mathematical reasoning take place (Hill et al., 2008). The difficulties of communication and of developing understanding through communication have been a major focus of research and continue to be a major topic of discussion and research (Pirie, 1998). The use of language by the teacher plays an important role in the ability of the students to create understanding of the mathematical content. The example given previously of the circumference lesson could also illustrate this point since the teacher failed to make clear the distinction between “pie” and “pi” during the lesson, since both words were part of the lesson. That the students might get confused by the use of both homonyms would be understandable. Teachers should have strong use of mathematical language and encourage and help their students to develop strong mathematical language use as well. Errors and imprecisions in the language could also affect the quality of the instruction and should be considered.

Stein, Grover, and Henningsen (1996) explore the idea of mathematical tasks given to students that maintain high cognitive demand. They define a mathematical task as “a classroom activity, the purpose of which is to focus students’ attention on a particular mathematical idea.” (p. 460) High-level cognitive demand tasks provide the basis for “building student capacity for mathematical thinking and reasoning”. High cognitive demand tasks are tasks that are given to students that cause them to think, reason and make sense of the mathematics in the classroom. Explanations, justifications, making connections, meaning-making, building on prior knowledge and engagement are examples of aspects of high cognitive demand tasks (Stein, Grover, & Henningsen, 1996; Stein & Smith, 1998). Explanations, making connections, multiple strategies, collaborative work, mathematical argumentation and error analysis also promote conceptual thinking (Hiebert & Grouws, 2007; Kazemi & Stipek, 2001). Tasks of this nature would be a part of instruction that promotes high cognitive demand and thus conceptual understanding for students.

Explanations are a high level cognitive demand task and an interaction that I feel should be considered when examining interactions in the mathematics classroom. Explanations should move beyond the recitation of procedures and of definition to explore the "why" behind the mathematics. Explanations might use a combination of both language and visual representations. Students, as they interact in the classroom, might also attempt to explain the mathematics. These explanations should be considered along with how the teacher uses them.

Research is also beginning to show that when teachers notice student thinking and integrate it into instruction that student learning outcomes are improved (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996; Kazemi & Franke, 2004; Stein, Grover & Henningsen, 1996) Teacher noticing of student thinking would be evident in teacher questioning and use of student contributions. Such use of student contributions would include use of student questioning, reasoning, explanations, attempts at meaning-making and even using and correcting incorrect student contributions. Whether the teacher uses the contributions and how the teacher uses them, would make a difference in student learning and the dynamics of the classroom (Kazemi & Stipek, 2001). The impact of an error by a teacher or student could be negligible or even beneficial depending on correction and/or use by the teacher. Even errors could be used to advantage in a mathematical discussion (Kazemi & Stipek, 2001).

In summary, I feel that the mathematical content and language are the core features of the interactions in the classroom. Considering the explanations and student contributions and how the teacher uses them are also important in studying the interactions in the classroom. These are aspects of the interactions I will consider when studying Finnish mathematics classrooms. I understand that these ideas come from U.S. research and theory and because of this come from a U.S. perspective but feel that studying these ideas will help to further our understanding of instruction in Finnish mathematics classrooms. I will attempt to answer the following questions: What are the mathematical interactions between teachers and students in the Finnish mathematics classroom like? Are the mathematical interactions of high or low quality with respect to cognitive demand? Are there any patterns to the interactions that might help in better understanding Finnish instruction?

Chapter 4: Methodology

In this section I will explain the methodology I used to answer my research questions. I will begin by explaining the setting and participants. Then I will explain the process of data collection. I will conclude with an explanation of how I analyzed the data.

Setting and Participants

I have done a video study of Finnish mathematics classrooms. The study is small, with 13 recorded lessons of which 8 were analyzed. Videos were collected in the Helsinki area and in a municipality in western Finland. The class levels varied with the majority being from 8^{th} or 9^{th} grade of peruskoulu, or the equivalent age of 9^{th} or 10^{th} grade in the United States. One video each of a 6^{th} and 7^{th} grade classrooms were also obtained and analyzed. Most math classes at the lower secondary level in Finland are for approximately 45 minutes, though in some schools the class time is as much as 90 minutes. At the elementary school level (1-6 grades) the math portion can be shorter. 5 of the 8 classes, that were coded, were about 45 minutes in duration. The class sizes for the different classes were under 25 students and most of them were under 20 students. An exact number of students in the classroom for some of the videos is not available.

Data Collection

I traveled to Finland and collected 8 videos from schools. I also was allowed access to an additional 5 videos collected by Enrique Garcia and Markku Hannula at the University of Helsinki. The videos I collected were obtained using a video camera and wireless microphone. The other videos were collected using a video camera. All appropriate permissions were obtained from parents, teachers, principals, and municipalities, where necessary. Videos from 8 different teachers were obtained. I also recorded a short interview with four of the teachers about their lessons. The questions I asked were: 1. Was this a typical lesson? 2 How do you feel the lesson went? 3. What went well in the lesson and why? 4. What would you like to improve in the lesson and why? 5. What is your process for preparing a lesson? The questions were used when selecting the videos to analyze, as well as, to see if any additional, insights could be gleaned about Finnish mathematics instruction.

Selection of the Video

A number of different factors were considered in selecting the videos for analysis. The quality of the sound and video was the biggest consideration in the selection of the videos and led to not including two of the videos. Not being able to see and hear the interactions taking place in the classroom would make it difficult to code and analyze the interactions taking place. The answers to the questions in the interview were also considered, whether or not the lesson was typical. Of the videos I recorded, I wanted to include at least one video of each teacher if possible. In one case I had 3 videos from one teacher. In order to give a more even distribution I selected two classes from this teacher, eliminating one of the two 9th grade classes for that teacher. This also helped give me an equal number of 8th and 9th grade classrooms in the final selection.

With the five videos obtained from the University of Helsinki, I chose to use only two of the videos, each from different teachers, and both from 9th grade. One of the biggest factors in selecting the videos was the ability to hear the interactions, since a teacher mic was not used. These classes also included the same research problem at the beginning of each class, for the research being conducted by Enrique Ortiz and Markku Hannula.

Included in the final selection of the videos, that were analyzed, are the only 6th grade class, the only 7th grade class, three 8th grade classes, and three 9th grade classes, with 6 different teachers being included in the videos coded. Six classes were videos of classes that I obtained and two were from the videos obtained from the University of Helsinki.

Analysis of the data

My study examined the interactions between teachers and students around the content in mathematics classrooms in the Finnish schools. I will describe the MQI (2013), which I used to analyze the videos, then explain how the MQI (2013) connects with my theoretical framework. I will also describe the translation and coding and analysis that I performed on the videos. For analysis of these interactions I used the *Mathematical Quality of Instruction* (MQI, 2013) instrument, which was already available. The *Mathematical Quality of Instruction* (MQI, 2013) instrument was designed by Heather Hill and associates at Harvard University to look at the type and quality of interactions of teachers and students around the content and the richness (or depth and quality) of the mathematics taught in the classroom. The instrument was created through a study of 250 U.S. videotapes of classroom instruction. The MQI codes include codes for aspects of the mathematics instruction and the interactions, like explanations, that I feel are important when studying the mathematics and mathematical interactions in the classroom. The MQI also codes for other aspects of instruction that promote high cognitive demand and conceptual understanding in students, for instance making connections, multiple strategies, engagement (enacted task cognitive activation), and meaning making, etc. For example, the richness of the mathematics codes include the linking of mathematical topics, the explanations of the why of the mathematics, the quality of the mathematical language and representations used by the teacher and students, and other aspects of instruction (See table 1).

The MQI (2013) code of Classroom Work is Connected to Mathematics indicates whether half or more of the interval coded is focused on the mathematical content. Work that is considered focused is teachers introducing and reviewing math and students working on math. Things that are not included are things like distributing materials, major disciplinary problems, and cutting and pasting activities. This code is either a yes, the majority of the interval was focused on mathematical content, or no, the majority of the interval was not focused on mathematical content.

Whole-Class Discussion looks at whether students build or reason based on another students contributions, or students interacting with students. A score of none would mean that students aren't building on each other's contributions while an all would mean for the whole segment students are building on each other's contributions and thus interacting with each other using the math content for the whole segment.

Table 1

MQI (2013) Categories and Segment Codes

MQI Category	Segment Codes
Richness of the Mathematics	• Linking and Connections • Explanations • Multiple Procedures and Solution Methods • Developing Mathematical Generalizations • Mathematical Language • Overall Richness of the Mathematics
Working with Students and Mathematics	• Remediation of Student Errors and Difficulties • Teacher Uses Student Mathematical Contributions • Overall Working with Students and Mathematics
Student Participation in Meaning-Making and Reasoning	• Students Provide Explanations • Student Mathematical Questions and Reasoning • Enacted Task Cognitive Activation • Overall Student Participation in Meaning-Making and Reasoning
Errors and Imprecision	• Major Mathematical Errors • Imprecision in Language and Notation (Mathematical Symbols) • Lack of Clarity in Presentation of Mathematical Content • Overall Errors and Imprecision
Other Segment Codes	• Classroom Work is Connected to Mathematics • Whole-Class Discussion • Students Work with Contextualized Problems • Students Communicate about the Mathematics of the Segment • Small Group/Pair Discussion • Mathematical Meaning and Sense-Making

Richness of the mathematics. The MQI (2013) code of Linking and Connections looks at whether teachers and students make explicit links and connections among representations, procedures, and mathematical ideas (See Appendix A). An example of a link or connection would be when the teacher shows how ratios and fractions are related. An example of what would not be considered a link or connection is a mention of the topic covered the day before. This code has three levels: low, mid, or high. Low is when links and connections between mathematical ideas, procedures, and representations are not made. If links and connections are made the score would be a mid or high depending on the whether the links are made with detail or explicitness. The MQI (2013) code of Explanations is different than perhaps would be commonly understood. Explanations in the context of the MQI include answers to the question of why, like "why a procedure works", "why a solution method makes sense", or "what a solution means" (MQI, 2013). Explanations can be offered by the teacher or students. Explanations do not include a recitation of the steps of a procedure or statements of fact or definitions without additional connections to the underlining meaning. The code has three levels: low, mid, and high. A score of low would mean that no mathematical explanations are given by the teacher or students. A score of mid would mean that there are explanations but they are not very detailed and are specific only to the current situation. A score of high would mean that the explanations are detailed and give meaning to procedures and/or solutions or that the explanations give meaning to the math of the problem as well as generalize to math beyond the problem. The MQI (2013) code of Multiple Procedures or Solution Methods assesses whether more than one solution method or strategy is given for a single problem by the teacher and/or students. The second solution does not have to be discussed fully. This code has three levels: low, mid and high, with a low being an absence of multiple solutions and a high being multiple solutions discussed with details like the advantages of a particular method or the efficiency of a particular method. A mid would be multiple solutions being given without much comparison of the methods.

The MQI (2013) code for Mathematical Language gives an indication of the fluency of mathematical language on the part of the teacher and whether the students are being taught to use mathematical language fluently. The three levels of this code are low, mid, and high. To obtain a score of high the teacher needs to use mathematical language frequently, fluently and correctly and have language specific interactions with the students, including actions like pressing students to use mathematical language and helping students to understand mathematical language, etc. A mid would be given when a teacher uses math language correctly throughout the lesson but does not include things like pressing the students for mathematical language use.

The category code of Overall Richness of the Mathematics in the MQI (2013) includes the codes of Linking and Connections, Explanations, Multiple Procedures or Solution Methods, Developing Mathematical Generalizations, and Mathematical Language (not included as a richness code in the 2014 version of the MQI). This code isn't an average of the codes in this category but gives an indication of the depth of the mathematics covered in the classroom. This code attempts to give an indication of whether students are given the opportunity to develop a conceptual understanding and understanding of mathematical practices and language. The Overall Richness of the Mathematics has four levels of low, low/mid, mid, and high and is an overall estimate of the richness of the mathematics during the segment. A low would mean that there are not any elements of richness. A low/mid would indicate there is an isolated mid score in this category and a mid for overall richness would indicate that there are some mid scores in this category. A score of high means there is a strong focus on aspects of richness that would lead to student conceptual understanding through connections, representations, generalization, solution methods, and language, etc. A high would also be scored if one or more of the codes is outstanding in its implementation in the classroom.

Working with students and mathematics. The MQI (2013) code of Remediation of Student Errors and Difficulties gives an indication of whether the teacher attempts to help students with misconceptions and difficulties in understanding the mathematical content. The remediation can happen individually with the students or in larger groups, including the whole class. There are three levels of this code, which are low, mid, and high. The coder looks at the type of remediation, conceptual or procedural, and the amount of the remediation to decide which score to give. A low score would mean that no effective remediation occurs or the remediation is both brief and procedural. A mid score is when the teacher engages in brief conceptual remediation or lengthy procedural remediation and a high score includes things like lengthy conceptual remediation, providing instruction that anticipates common student errors and/or addressing how the student errors are part of a larger problem and attending to that as well.

The MQI (2013) code of Teacher Uses Student Mathematical Contributions helps us understand how the teacher uses student contributions during class time. Contributions by the student include answers to the teacher's questions, comments on the lesson, explanations of ideas, representations of the math, other student work, questions asked by the students, etc. The four levels for this code are not present, low, mid, and high. Not present is used when the students contribute very little or the teacher responds in a way that confuses the lesson. Low means that the teacher responds to the students in a pro forma way by saying only something like "yes" then moves on with the lesson. With a score of mid, the teacher moves beyond a pro forma response and uses them a little in the lesson but fails to mesh the ideas into the development of the lesson like with a high score. A high in this category would mean the teacher uses the student contributions extensively and may include assigning ownership of ideas to students and highlighting aspects of a student's contributions or questions.

Overall Working with Students and Mathematics is not meant to be an average of the codes in this category but is meant to give an indication of the amount of substantive interactions between the teachers and students with relation to the content. This is a 4 point code, with low, low/mid, mid, and high. With each level the degree of interactions between teacher and students around the math increases. A low score would mean that there aren't any significant interactions between the teacher and students or if there are substantive contributions the teacher fails to respond well to them. A high would mean that the teacher responded outstandingly to student errors and uses student ideas in a strong manner, which would be seen by the amount of mid and high scores in the sub codes of this category.

Errors and imprecisions. The MQI (2013) code of Major Mathematical Errors is an indicator of the errors made by the teacher with respect to the mathematics. Mathematical errors would include incorrect mathematical definitions, not solving a problem correctly, making major errors in written or spoken work. A score of low would mean the teacher made no errors or noticed and corrected any errors within a short time of making them. A mid score would be given with few errors of shorter duration or forgets important aspects of the mathematical situation. A high score would indicate the teacher making major or many errors in their spoken and/or written work or failing to important aspects of the content with respect to the problems given.

The MQI code of Imprecision in Language or Notation is similar to Major Mathematical Errors but primarily deals with errors in the language and notation. Errors in the symbols, spelling, use of words incorrectly or errors in general language used to describe mathematical ideas. This code has three levels: low, mid and high. A low would be a lesson in which the teacher was clean of these types of errors. A teacher who makes small, brief errors of this type would receive a mid. A high would be characterized by sloppiness in notation and language or major errors in notation and language.

Lack of Clarity of Presentation of Mathematical Content gauges whether parts of the lesson are confusing or distorted because of errors or poor explanation. A poor launch of an activity would also count as a lack of clarity. This code also has three levels of low, mid and high. A low score would indicate that the lesson was clear and understandable. A high in this code would mean that the lesson was unclear, incomplete, or distorted for a significant amount of the segment scored. A mid would be given if there was lack of clarity but only for a smaller portion of the segment.

The code of Overall Errors and Imprecision gives an indication of the amount of errors and imprecision in the lesson. There are four levels to this code: low, low/mid, mid, and high. A low would mean that no errors occurred during the lesson. A low/mid would be "small, momentary error(s)" or a "brief lack of clarity". A high would be multiple errors, a big error, or lack of clarity throughout the segment. A mid lies between a low/mid and a high meaning the errors and lack of clarity were neither momentary nor were they many and throughout the whole segment.

Student participation in meaning-making and reasoning. The MQI (2013) code of Students Provide Explanations is similar to Explanations but looks specifically at the explanations given by the students. Like with explanations the student has to explain "what an answer means", etc. The explanations given by the students can be co-constructed with the teacher and also do not have to be complete or correct. Since a student explanation does not have to be complete or correct, you can have a student explanation without it counting under explanations. Additionally the student can also compare the benefits of different methods. This code has three levels: low, mid, and high. A low is an absence of student explanations and a high includes explanations that generalize beyond the problem being considered. A mid would be explanations that are specific only to the problem being worked on.

The Student Mathematical Questioning and Reasoning code gives an indication of instances of student mathematical thinking, like counter-claims, mathematically motivated questions, conjectures, examples, hypothesizing, and connections to other mathematical topics. Like with Students Provide Explanations the contributions don't need to be complete or correct. This code also has three levels of low, mid and high. A mid would indicate there are one or two examples of student questioning and reasoning and a high would indicate that there are three or more examples of student questioning or reasoning.

The MQI (2013) code of Enacted Task Cognitive Activation looks at the degree to which the students engage with the mathematics in the lesson. Are they doing routine activities or thinking more deeply about the math through making connections, conjectures, justification and other deeper thinking activities? This code has three levels: low, mid and high. A low involves low cognitive demand activities like listening to the teacher lecture or recalling known procedures, etc. The mid on this score includes cognitive activation that is high at some points but low at other points but can also be scored for substantive student interactions at different points in the lesson. A high in Enacted Task Cognitive Activation includes things like drawing connections, looking for patterns, and/or justifying, etc.

The Overall Student Participation in Meaning-Making and Reasoning code gauges the level of involvement for the students in doing mathematics and contributing to meaning-making and reasoning in the class. Activities that are considered are things like student explanations, question-asking, and reasoning or student participation in small group or pair work on a non-routine task. This code has four levels: low, low/mid, mid, and high. A low in this code would mean that there aren't any good examples of these type of activities. A high would indicate that such student contributions are a major feature of the lesson or that the students are working on a challenging task. The mid score has some aspects of the students demonstrate the behaviors of a high but not as strongly nor for as long. I might ask myself if the students are engaged and doing things like asking questions or reasoning about the math throughout the segment.

Other segment codes. The MQI (2013) code of Students Work with Contextualized Problems attempts to indicate whether problems worked on in class are contextualized, ie. real-world applications or statistics, or story problems. The difficulty with this code is when there isn't any indication of the quality or level of the tasks given during classwork because a worksheet or problems from the book are assigned. This code has four levels: not present, low, mid, and high. Not present would mean that as far as the coder knows no contextualized problems are worked on in class. A high would mean that contextualized problems are worked on in detail with the students doing most or all of the thinking and reasoning.

Students Communicate about the Mathematics of the Segment looks at the frequency of all substantive student mathematical contributions in a segment. Substantive contributions include things like asking mathematical questions, students sharing their mathematical work (written or verbal), discussing meanings and methods, explanations, etc. This code is similar to Overall Student Participation in Meaning-Making and Reasoning, but there are some differences in what is being included. This code also only applies when contributions are given in groups or to the whole-class. Contributing in pair work or when talking individually with the teacher do not count under this code. There are four levels to this code: not present, low, mid and high. A high would indicate that throughout the majority of the segment there are substantive contributions.

Small Group/Pair Discussion attempts to gauge the amount of student discussion of mathematics in small group and pair work. However, it is difficult to hear the interactions among students especially when there isn't a microphone specifically for them, which was the case in this study. This was an experimental code in the MQI (2013) and was dropped in the next iteration of the MQI.

The MQI (2013) code of Mathematical Meaning and Sense-Making indicates whether the teacher and students address ideas like the meaning of and relationship between numbers, understanding the context associated with the numbers, whether the solution method and answer to a problem makes sense and making connections between mathematical ideas and representations, etc. This code has four levels: not present, low, mid, and high. A low would mean that there is only brief attention to mathematical meaning and sense-making by either the teacher or the student. A high would mean that meaning making can be found throughout the segment and is the main point of the instruction.

Theoretical framework and the MQI. The reason for selecting the MQI as an analysis tool is that many of the codes of the MQI (2013) can be connected to my theoretical framework and thus help understand my research questions about the interactions and cognitive demand in the classroom. In my theoretical framework I talk about the interactions between teacher and students, the mathematical content, the mathematical language, the cognitive demand, explanations and noticing student thinking. I will discuss how these ideas relate to the MQI (2013).

A number of the MQI (2013) codes are related to interactions in the classroom. Whole-Class Discussion shows the presence of student discussion and reasoning based on another student's contributions. Remediation of Student Errors and Difficulties looks at a specific type of interaction between teachers and students, whether the teacher is helping students with their difficulties through remediation. Teacher Uses Student Mathematical Contributions gives an indication of how a teacher is using student contributions, whether the teacher is not using them or responding to contributions in a pro forma way or weaving the student contributions into the lesson. Overall Working with Students and Mathematics is meant to show whether the amount of substantive student interactions. These and other codes will help me to understand the amount and type of interactions between the teacher and students.

The use of mathematical language and notation used in the classroom are addressed in the MQI (2013) codes of Mathematical Language, Imprecision in Language or Notation, and Lack of Clarity in Presentation of Mathematical Content. Mathematical Language indicates the fluency of the mathematical language of the teacher as well as whether the teacher explains the language and presses the students to use the language. Imprecision in Language or Notation addresses whether any errors are made in language and notation. Lack of Clarity in Presentation of Mathematical Content indicates whether the teaching has become confused because of teacher errors in math, language or lack of explanation.

Cognitive demand deals with the tasks and activities in the classroom and is addressed in the MQI with a number of the codes. These tasks and activities are not only what the students do but the reasoning activities that occur in the classroom like explanations, making connections to other material, using multiple strategies, working together, and meaning-making activities, etc. (Stein, Grover, & Henningsen, 1996; Stein & Smith, 1998). A number of the MQI (2013) codes look at these types at activities. Explanations and Student Explanations address the cognitive demand task of explaining the meaning of the mathematics and the why of the mathematics. The Linking and Connections code indicates whether connections are being made to other mathematics outside the lesson, which is another cognitive demand task. Student Mathematical Questioning and Reasoning code looks at whether the students justify, ask mathematically motivated question, give counter-claims, and hypothesize, etc., which are cognitive demand activities. There are also other codes that could help to understand the cognitive demand, for instance, Overall Student Participation in Meaning-Making and Reasoning, which helps to understand whether students are participating in meaning-making and reasoning activities.

Noticing of student thinking is also shown through various codes of the MQI (2013). These codes are Teacher Uses Student Mathematical Contributions, Overall Working with Students and Mathematics, and the whole lesson codes (see Table 4) of Teacher Uses Student Ideas and Teacher Attends to and Remediates Student Difficulty. Teacher Uses Student Mathematical Contributions indicates by varying degrees how much the teacher integrates student thinking into the lesson. Overall Working with Students and Mathematics is not as strongly indicative of student idea use but indicates the amount of interactions between the teacher and students including using student ideas in the lesson. Teacher Uses Student Ideas is a whole lesson code discussed later that gives a score, 1 - 5, of how true the following statement is for the lesson "Teacher uses student ideas and solution to move the lesson forward". Teacher Attends to and Remediates Student Difficulty is another whole lesson code. The highest score of 5 can be achieved by the teacher "understanding and addressing the root of student difficulty" which is generally achieved by noticing student thinking to begin with.

The MQI instrument has been used at the elementary and middle school levels to assess instruction in U.S. Although the MQI was created from analyzing U.S. classrooms and is being used to analyze U.S. classrooms, I feel using the MQI could still give valuable insight into the interactions of Finnish mathematics classrooms from a U.S. perspective.

Coding of the videos. The videos of the Finnish mathematics classrooms were transcribed by a non-native Finnish speaker and a native Finnish speaker. The translation was done by a native Finnish speaker and a native English speaker working together. The lessons were reviewed and the selection of the videos, that were coded, was based on the interview with the teacher at the close of the lesson, the quality of the video, and the amount of mathematical interactions between the teacher and students (see Appendix B). Each lesson that was selected was coded by two coders using the MQI in seven and a half minute segments. Differences in coding were present and some discussion of these differences occurred. In the end, I reviewed the reasons for the codes and made a decision on the coding where differences were present. After the coding for each video was completed and decisions made, I took an overall average for each of the codes for each lesson.

The inter-rater reliability for the segment codes for all the videos was approximately 83%. The video with the least agreement was A1 with a 76% and the most agreement was video A5 with 91%. An average of the combined codes was also taken for all the segments for each video. Most of the videos had six sections though one had five, one had seven and another had eight sections. For the segment codes, with three levels of low, mid, and high, a low was given a one, a mid was given a 2, and a high was given a 3. For the four level segment codes, which had levels of low, low/mid, mid, and high, four point values were assigned. A low was a one, a low/mid was a 1.5, a mid was a two, and a high was a 3. The scores were decided on after consulting with the MQI master coders since that would facilitate possible, future comparisons of results of other MQI based studies. For the four level segment codes which had levels of not present, low, mid, and high, points were assigned as follows: not present – 0, low – 1, mid – 2, high – 3. For example, an overall score of 1.33 would indicate that, for at least one segment, a code level above a low was assigned.

The averages were used to help describe the classrooms, for instance, the amount of time spent on instruction and the types of student and teacher interactions as assessed by the MQI. I assessed if there are many student contributions and the degree to which the teacher uses the student contributions. The coding also gave me an indication of the presence and amount of high quality interactions between the teacher and students, characterized by such things as explanations, engagement, student thinking, and meaning making.

As I coded and analyzed the video I looked for any patterns or trends across the lessons which would help me to describe a possible typical lesson in a Finnish mathematics classroom. Notes had been taken while visiting the classroom and memos were used during the coding and analysis to record any aspects of the classroom that might be interesting or useful.

Using this information I will answer my three questions: What are the mathematical interactions between teachers and students in the Finnish mathematics classroom like? Are the mathematical interactions of high or low quality with respect to cognitive demand? Are there any patterns to the interactions that might help in better understanding Finnish mathematics instruction?

Chapter 4: Results

In this section I will describe the classroom, what a typical lesson is, based on the videos, and give two examples of lessons. I will also describe some of the MQI codes and what the analysis showed for these codes. I will talk about some of the things that I discovered from the interviews. I will also share some of my observations. Then conclude with how my results answer my three research questions.

The Classroom

The classroom arrangement in all the classes was typical of what you would find in a U.S. secondary classroom. The chalkboards were in the front of the class along with the teacher's desk off to one side. At the front of the class was a document camera and LCD projector with pull-down screen, which the teacher could use to project notes or student work. The LCD projectors could also link to the teacher's computer or to I-pads that were used in a few of the classrooms. Unlike Seaberg (2015), I did not see any interactive whiteboards. I did see use of programs like Kahoot (an on-line quiz application) and Geogebra (a free graphing utility), which were used by students working in pairs, on i-pads. The floors were bare of carpet and the walls were also fairly clear of decoration, though one classroom had a display of geometric designs created by students. This lack of decoration could possibly be because more than one teacher may teach in a classroom. The classrooms in the lower grades had more things on the walls, for instance pictures and explanations of the different mathematical symbols or students' work. In many ways, the classrooms appeared similar to U.S. secondary classrooms.

The classroom also had a relaxed atmosphere, which was also noted by Seaberg (2015). This relaxed feel could have been due to a number of factors. A factor that might contribute to this atmosphere are the fact that students take most or all of their classes together and often have the same teacher for subjects, like math, for several years consecutively. Other factors that might contribute are the 15 minute breaks between their classes or that students don't have to wear shoes in class.

Typical Lesson

Although there were differences to be found among the videos, seven of the thirteen lessons recorded followed essentially the same format. Four of the eight videos analyzed had time spent on review as well. The lessons typically had four parts: homework review, instruction on new material, in-class work time, and the assigning of homework before the end of class. I will explain in more detail what is meant by each part.

The homework review, when done, was the first part of the lesson. This was the part of the lesson that was most likely to not be included. However, 8 of the 13 lessons recorded included this as part of their class time. The teacher would select a few problems, typically three, from the homework to review. The students would then volunteer to write their work on these problems on the board or show their work using a document camera. The teacher would then review the problems by talking through the steps and correcting the work as necessary. In only one class of the thirteen videos did the students, instead of the teacher, talk about their work. And in only one class of the eight analyzed did a student's work need to be corrected.

For most of the lessons, the second segment of class time was instruction on new material. In two of the classes analyzed this part of the lesson was re-teaching of material or reviewing material on a test. This part of the lesson had variable amounts of interaction between the teacher and students. However, in almost all of the classes, students seemed comfortable and willing to volunteer answers or ask questions. The instruction included relevant definitions, if needed, and typically around three examples of problems related to the new material. The lesson segment was generally about 20 minutes in length. The sixth grade class, however, had more examples and activities throughout the class with the teacher drawing the students back into instruction at various points throughout class time.

The last major portion of the lesson was time for students to work on problems related to the new material in class. Students would work in pairs or individually while the teacher walked around the class answering student questions. In most of the classes the students were engaged in math related work until the end of class time, which in the videos analyzed averaged about 10 minutes. The answer key was also readily available in many of the classes to help the students assess their work. At the conclusion of this time, the end of class, homework was assigned with little or no explanation on the problems assigned.

Lesson Example 1

My first lesson example is a ninth grade class with 11 students, 7 girls and 4 boys. The teacher started class by asking students to put the three different parts of a homework problem on triangular prisms on the board. Three students wrote their work on the board, one student for each part. The problem is not written on the board just the student work so the wording of the original question is unknown. Part A, which was to find the volume of the triangular prism, was done incorrectly by the student. The student had found the area of the rectangle on the lateral face and multiplied by the height of the triangle of the base. The teacher remediated the errors and used a 3-D triangular prism as well as drew an illustration on the board to help describe the faces and calculations needed. The teacher explained that you need to find the area of the base, in this case the triangle and then multiply that by the height of the prism. After explaining the formula, the teacher then calculated the volume of the prism and converted the metric units from mm^3 to cm^3.

The teacher then moved on to Part B of the problem, which was written on the board by another student. Part B of the problem was finding one of the sides of the triangle using the Pythagorean Theorem and then calculating the area of the lateral, rectangular faces. The student work was correct but the teacher still talks through the work on the board. The teacher explained the formula that was used, while showing on the illustration of the triangular prism the parts used in the formula. The teacher also quickly reviewed the student's calculations. Then part C (work by another student) was reviewed by the teacher, which is the calculation for the surface area of the whole triangular prism.

After reviewing the homework problems the class transitioned to a lecture on new material, which was finding the surface area of a cylinder. Finding the volume of a cylinder was done in a previous lesson. The students took notes in their school-provided notebooks of what the teacher wrote on the board. 3-D cylinders were used to introduce the formula for finding the surface area of the problem. The teacher used the cylinders to show the different parts of the cylinder and demonstrates that when the lateral side is cut and laid flat it has the form of a rectangle. The teacher asked the students for the formulas to calculate the area of a circle and of a rectangle and then showed how the formulas correspond to the different parts of the cylinder. Using these formulas the teacher then did an example to show how the formulas for the area of a circle and the area of a rectangle can be used to find the surface area of the cylinder (see figure 3). When the examples were complete, the teacher gave the classwork. For the remaining class time, the students worked alone or in pairs while the teacher walked around and answered questions individually. At the end of the class the teacher gave the homework assignment.

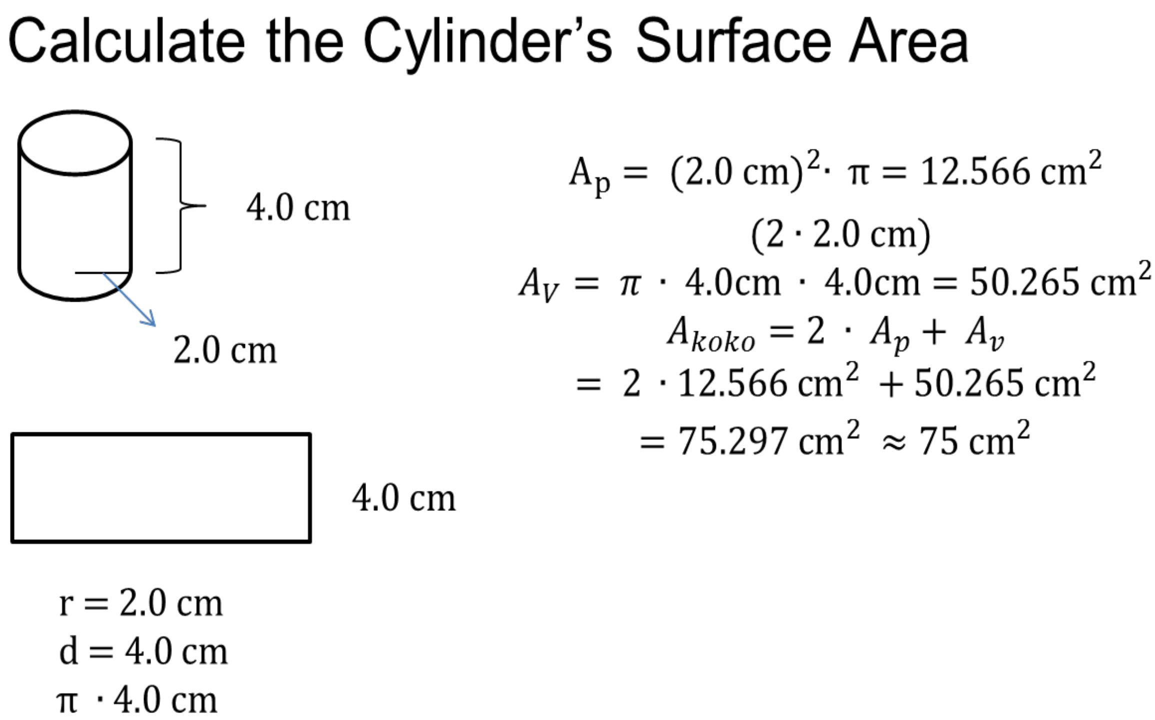

Figure 3. Calculate the Cylinder's Surface Area Boardwork

Lesson Example 2

My second lesson example is a seventh grade class with about 20 students. At the beginning of the class, the teacher asked several students to write their work from the homework on a sheet of paper, so that the work could be projected on a document camera. While the students were writing down their work, the teacher walked around and checked the students' completion of homework. The student work, which was on product and quotient rule of exponents, was then reviewed by the teacher with the whole class. One student shared the following work: $\frac{k^5 \cdot k^4}{k \cdot k^6} \cdot k^8 = \frac{k^{5+4}}{k^{1+6}} \cdot k^8 = k^{9-7+8} = k^{10}$. The teacher then added an alternative ending to the problem $\frac{k^9}{k^7} \cdot k^8 = k^2 \cdot k^8 = k^{10}$. The homework review took about 13 minutes.

After the student work was reviewed, the teacher began a lesson on the power rule for exponents, $(a^n)^m = a^{n \cdot m}$. The teacher then gave the following expression $(x^2)^3$ and asked the students to use their prior knowledge and suggest another way to write the expression. One student suggested $x^2 \cdot x^2 \cdot x^2$. The teacher then asked for another suggestion and a student suggested $x \cdot x \cdot x \cdot x \cdot x \cdot x$. The teacher then asked what the answer would be and a student responded x^6. On the overhead the teacher had written $(x^2)^3 = x^2 \cdot x^2 \cdot x^2 = x \cdot x \cdot x \cdot x \cdot x \cdot x = x^6$. Then the teacher gave the students the power rule, $(a^n)^m = a^{n \cdot m}$. After sharing the rule, the teacher did one more example, $(n^3)^2$, then gave three problems for the students to work on, $(c^5)^{10}$, (x^{100}), 3^{2^3}. They then reviewed the problems as a class. The lesson took about eleven and a half minutes.

After the lesson, the students were given more classwork out of the book. While they worked on this quietly, the teacher walked around and helped students as they need on the classwork. The students were focused and worked on the classwork for the rest of the class. The teacher gave the homework at the end of class time.

MQI Codes and Analysis

In this section I will talk about the MQI (2013) codes I used and the results obtained through coding using the MQI. For the three level segment codes a low was given a one, a mid was given a 2, and a high was given a 3. For the four level segment codes which had levels of low, low/mid, mid, and high, four point values were given. A low was a one, a low/mid was a 1.5, a mid was a two, and a high was a 3. For the four level segment codes which had levels of not present, low, mid, and high, points were assigned as follows: not present – 0, low – 1, mid – 2, high – 3.

The MQI (2013) code of Classroom Work is Connected to Mathematics indicates whether half or more of the interval coded is focused on the mathematical content. All of the videos, but one, had a "yes" in all of the sections, which means the majority of each segment was focused on mathematical content (see table 2; see Appendix C). In the one video that did not have all sections marked "yes", two sections received a "no" because the teacher took a little more than three and a half minutes to switch from one activity to another. This delay was in one of the research lessons. The delay was partially due to the teacher asking the researcher what else was required for the research. The other section that was marked "no" was for the final 3 minutes of class time. Classroom work in all of the classes was connected to mathematics for nearly all of class time.

Richness of the mathematics. The MQI (2013) code of Linking and Connections looks at whether teachers and students make explicit links and connections among representations, procedures, and mathematical ideas (see Appendix A). A high would be when strong or detailed links and connections between mathematical ideas, procedures, and representations are made. Four of the classes had connections that were made in at least one coded section, though none of the connections rose above a mid. As shown in table 2, the lowest overall average is 1.00 meaning no links and connections were made or none were significant. The highest overall average is 1.8, for class A1. Four connections were made in four of its five segments though none of the connections rose above the level of a mid (see Table C1).

The most common connection made in the classes was between the numbers and representations, through graphs or physical models, to help students understand the relationship between the numbers and the context. One example of a link or connection made was as the teacher was talking about the area of a rectangular shape, as he indicated on the drawing where the different measurements came from. He also indicated each of the individual squares that made up the total area. Another example of a link or connection made was the use of a 3-D triangular prism by the teacher as she reviewed a student's work on surface area that was already written on the board. The student had made some errors in calculating the surface area because of a lack of understanding of how many and of what type of surfaces to include in the calculation. As the mistake was remediated, the teacher referred to a drawing on the board as well as the 3-D representation of a triangular prism.

Table 2

Overall Lesson Averages for Richness of the Mathematics and Working with Students and Mathematics

MQI Codes	A1	A2	A3	A5	A6	A8	C1	C2	Avg
Number of Segments	5	6	6	6	6	6	8	7	
Classroom Work Connected to Mathematics	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.71	0.96
Richness of the Mathematics									
Linking and Connections	1.80	1.00	1.33	1.00	1.33	1.33	1.00	1.00	1.22
Explanations	1.20	1.50	1.00	1.00	1.17	1.33	1.13	1.00	1.17
Multiple Procedures or Solution Methods	1.00	1.33	1.00	1.00	1.00	1.33	1.00	1.00	1.08
Developing Mathematical Generalizations	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Mathematical Language	1.80	2.00	2.17	2.00	2.00	1.83	2.00	1.86	1.96
Overall Richness of the Mathematics (4-point scale)	1.70	1.33	1.33	1.00	1.25	1.67	1.06	1.00	1.29
Working with Students and Mathematics									
Remediation of Student Errors and Difficulties	1.40	1.67	1.33	1.67	1.67	1.50	1.25	1.29	1.47
Teacher Uses Student Mathematical Contributions (4-point scale)	1.40	1.50	1.17	0.67	1.67	1.83	1.63	1.29	1.40
Overall Working with Students and Mathematics (4-point scale)	1.30	1.83	1.33	1.67	1.83	1.83	1.56	1.36	1.59

As stated previously, the MQI (2013) code of Explanations is different than perhaps would be commonly understood. Explanations in the context of the MQI include answers to the question of why, such as “why a procedure works”, “why a solution method makes sense”, or “what a solution means” (MQI, 2013). Explanations can be offered by the teacher or students. Explanations do not include a recitation of the steps of a procedure or statements of fact or definitions without additional connections to the underlining meaning. Five of the videos included explanations of this type, mostly given by the teachers (see Table 2, Appendix C). An example of an explanation given is when the teacher explained that $2^3 = 8$ because 2^3 is $2 \cdot 2 \cdot 2$. Another example of an explanation is when the teacher asked why you could replace the diameter with two times the radius in the formula for the area of a circle. A student explained that the diameter equals two times the radius so that is why you can replace the diameter, d, with 2r in the calculations for the circumference.

The MQI (2013) code of Multiple Procedures or Solution Methods assesses whether more than one solution method or strategy is given for a single problem by the teacher and/or students. Only two classes had evidence of multiple solutions and both had an overall score of 1.33 for Multiple Procedures or Solution Methods (see Table 2). The first example was a teacher who discussed two different ways to calculate a problem on percentages using a bar graph. Using the bar graph also illustrated how the numbers related to each other. The second example is when a teacher gave an alternate path to the solution on a homework problem. A student presented the following: $\frac{n^8 \cdot \frac{n^4 \cdot n^5}{n \cdot n^2}}{n^3 \cdot n^4} = \frac{n^8(n^{9-3})}{n^3 \cdot n^4} = n^{14-7} = n^7$. The teacher shared this alternate path: $\frac{n^8 \cdot \frac{n^4 \cdot n^5}{n \cdot n^2}}{n^3 \cdot n^4} = \frac{n^8 \cdot \frac{n^9}{n^3}}{n^7} = \frac{n^8 \cdot n^6}{n^7} = \frac{n^{14}}{n^7} = n^7$.

The MQI (2013) code of Mathematical Language gives an indication of the fluency of mathematical language on the part of the teacher and whether the students are being taught to use mathematical language fluently. All of the teachers consistently used mathematically correct language throughout their lessons with the only reduction in mathematical language occurring in the end of class as students completed their classwork, which was the case with all the overall scores lower than 2.00 (see Table 2). Except in two classes, there wasn't a lot of pressing for students to use mathematical language or helping students to understand mathematical language since the students already seemed to understand the mathematical language. In the sixth grade class, which had the highest average for this code, a score of 2.17 (see Table 2), the lesson began with a discussion of what area and perimeter are and how they are different and then the lesson continued with further exploration of area and perimeter of 2-D shapes.

The category of Overall Richness of the Mathematics in the MQI (2013) has four score levels (low, low/mid, mid, and high) and includes the codes of Linking and Connections, Explanations, Multiple Procedures or Solution Methods, Developing Mathematical Generalizations, and Mathematical Language. These codes together are to help assess the depth of the mathematics covered in the classroom and to answer the following question: Are students being given the opportunity to develop a conceptual understanding and an understanding of mathematical practices and language? In the next iteration of the MQI, the MQI master raters decided that an isolated mid in mathematical language would not raise the level of overall richness. I decided to follow this rule in this coding as well. Since mathematical language would not raise the overall richness of the mathematics, two of the videos scored 1's in every segment for Overall Richness of the Mathematics (see Table 2, See Appendix C). Another video had only one section with a low/mid. All other videos had at least two sections with richness scores greater than 1. Looking at the overall lesson averages, the two strongest lessons for Overall Richness of the Mathematics had scores of 1.70 and 1.67. These were lessons A1 and A8, respectively.

Working with students and mathematics. The MQI (2013) code of Remediation of Student Errors and Difficulties gives an indication of whether the teacher attempts to help students with misconceptions and difficulties in understanding the mathematical content. Remediation occurred in all of the classes during classwork time though teachers took time to remediate during homework review time and lesson time as well (see Table 2, Appendix C). Most of the remediation in the classes was procedural but there were some examples of conceptual remediation, for instance, when the teacher used a 3-D model to explain the surface area formula of a triangular prism. In another class the teacher and students spent a considerable amount of time toward the end of class helping a student who was struggling with understanding the relationship between the diameter and the circumference of a circle. The teacher even got a cup and a piece of string for the student to explore the relationship with.

The MQI (2013) code of Teacher Uses Student Mathematical Contributions helps us to understand the teacher use of student contributions, like answers to the teacher's questions, comments on the lesson, explanations of ideas, representations of the math and other student work, and questions asked by the students, etc. A score of 0 for not present is possible in this code. While much of the use of student contributions was pro forma overall, there were some higher forms of use through integrating students' comments into the instruction. Scores higher than 1.00 would mean that, for at least one segment during class time, students' comments and questions were used beyond simple acknowledgement by the teacher (See Table 2). Seven of the eight videos had scores higher than 1.00. The highest overall score was A8, which had mid-level use in several segments (see Table 2, Table C6). The lowest overall score for this code was class A5 with a score of 0.67 (see Table 2, Table C4). This was the only class with a score below 1.00. For A5, which had a score of 0.67 there was one segment in which there was some disagreement between the coders, with one coder marking a low and the other a mid. The reason for the difference was confusion over the students' comments and questions which were at times difficult to hear making the translation difficult. The mid was given by one rater because of the replies by the teacher, even though the student comments and questions were difficult to transcribe and translate.

Overall Working with Students and Mathematics is meant to give an indication of the amount of the substantive interactions between the teachers and students with relation to the content. Three classes had an overall average of 1.83 (See Table 2). The lowest overall average was 1.30. This indicates that better quality interactions during instruction or classwork time occurred in every class.

Errors and imprecisions. The MQI (2013) code of Major Mathematical Errors is an indicator of the teacher's mathematical errors like incorrect mathematical definitions, not solving a problem correctly, making major errors in written or spoken work. A score of 1.00 means that no major mathematical errors occurred. The highest overall score for this code was 1.20 and the low was 1.00, which is the lowest possible score. Six of the eight classes had an overall score of 1.00 (see Table 3). For the errors in the two lessons there was disagreement about whether the errors should be considered errors or imprecision. In the end it was decided to record them as errors. The lesson, A1, that had the 1.20, had one error, raising that segment to a mid (See Table 3, Table C1). The teacher was explaining circumference and mentioned that circumference could be used to find the area of the crust on a pizza. This lesson only had 5 sections so the weight of the one error when averaging was greater than it would be in other lessons. The other lesson, A3, which had an error score of 1.17, had six sections (see Table 3, Table C3). There were two small errors in one section, so the section was given a mid. The first error was the endorsement of the area as the length times the width, after saying the area is the amount of space enclosed. The other error was mentioning that an example of area would be the amount of sand in a volleyball pit.

The MQI code of Imprecision in Language or Notation is similar to Major Mathematical Errors but primarily deals with errors in the language and notation. There was only one lesson in which imprecision was noted (See Table 3, Table C7). The imprecision was the failure to write the squared for the expression $(5y + 1)^2$. Since the teacher said the squared and then found the product $25\, y^2 + 10\, y + 1$, the imprecision in notation appeared to go unnoticed by the students.

Lack of Clarity of Presentation of Mathematical Content gauges whether the parts of the lesson are confusing or distorted because of errors or poor explanation. A low score, scored as a 1.00, would indicate that the lesson was clear and understandable. All the lessons received an overall average score of 1.00 for lack of clarity, meaning all the lessons were clear and not confusing (see Table 3).

The code of Overall Errors and Imprecision gives an indication of the amount of errors and imprecision in the lesson. A low, with a score of 1.00, means there are no errors and imprecisions and that the lesson was clear in its presentation. No highs were scored for any of the segments in any of the classes. Five of the videos did not have any errors or imprecisions at all and so scored a 1.00 (see Table 3). Three of the classes had one segment in which errors or imprecisions occurred but over the whole class these were quite small yielding an overall average for the classes of 1.10, 1.29, and 1.06.

Table 3

Overall Lesson Averages for Errors and Imprecisions, Student Participation in Meaning-Making and Reasoning, and the New Segment Codes

MQI Codes	A1	A2	A3	A5	A6	A8	C1	C2	Avg
Number of Segments	5	6	6	6	6	6	8	7	
Errors and Imprecision									
Major Mathematical Errors	1.20	1.00	1.17	1.00	1.00	1.00	1.00	1.00	1.05
Imprecision in Language or Notation	1.00	1.00	1.00	1.00	1.00	1.00	1.13	1.00	1.02
Lack of Clarity in Presentation of Mathematical Content	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Overall Errors and Imprecision	1.10	1.00	1.29	1.00	1.00	1.00	1.06	1.00	1.06
Student Participation in Meaning-Making and Reasoning									
Students Provide Explanations	1.20	1.50	1.00	1.00	1.00	1.00	1.25	1.00	1.12
Student Mathematical Questioning and Reasoning	1.20	1.17	1.67	1.00	1.67	1.33	1.88	1.14	1.38
Enacted Task Cognitive Activation	1.20	1.17	1.50	1.00	1.17	1.17	1.63	1.43	1.28
Overall Student Participation in Meaning-Making and Reasoning	1.20	1.42	1.50	1.00	1.17	1.17	1.38	1.14	1.25
New Segment Codes (4-point scale)									
Students Work with Contextualized Problems	0.60	0.00	0.67	0.50	0.33	0.00	0.00	0.00	0.26
Students Communicate about the Mathematics of the Segment	0.80	1.33	1.33	0.33	1.33	0.67	1.63	1.00	1.05
Small Group/Pair Discussion	0.20	0.00	0.00	0.17	0.17	0.33	0.75	0.14	0.22
Mathematical Meaning and Sense-Making	1.40	0.83	1.33	0.33	0.67	0.83	0.88	1.00	0.91

Student participation in meaning-making and reasoning. The MQI (2013) code of Students Provide Explanations is similar to Explanations but looks specifically at the explanations given by the students. The explanations given by the students can be co-

constructed with the teacher and also do not have to be complete or correct. A low, 1.00, is an absence of student explanations and a high includes explanations that generalize beyond the problem being considered. Only three classes had examples of student explanations (see Table 3). One of them is an example, co-constructed with the teacher, which has already been given, of what $(x^2)^3$ means.

As stated previously, Student Mathematical Questioning and Reasoning seeks to identify instances of student mathematical thinking, like counter-claims, mathematically motivated questions, conjectures, examples, hypothesizing, and connections to other mathematical topics. All of the videos, but A5, had at least one segment that scored a mid for this code (see Table 3, Appendix C). A5 received lows in all its segments, so had an overall average of 1.00 (see Table 3, Table C4). Several of the videos had more than one segment that received a mid-rating. A3 and C1 were the strongest in this area as indicted by the overall averages of 1.67 and 1.88, respectively. C1 received at least a mid in 6 of the 8 segments and had one segment that scored a high (See Table C7). The high was achieved as students discussed the placement of lines to enclose an area. Students gave different possibilities and reasoning for the placement of lines, etc.

The MQI (2013) code of Enacted Task Cognitive Activation looks at the degree to which the students engage with the mathematics in the lesson. Are they doing routine activities or thinking more deeply about the math through making connections, conjectures, justification and other deeper thinking activities? This code is easier to discern during active, instruction segments. It is harder to know what is required of the students when they are given work to complete from the book. An overall score of 1.00 would indicate that there were no discernible activities that engaged the students to think at higher levels. Only one of the eight classes, A5, scored 1.00 (See Table 3). All of the other classes had segments with cognitive activation. The highest score was for lesson C1. This class began with work on the research question from Enrique Ortiz. The teacher asked for suggestions for solutions and then led the class in discussion of which solution was better. He promoted discussion, hypothesizing and justifying of hypotheses. Unfortunately, it was difficult to hear student responses at times.

Overall Student Participation in Meaning-Making and Reasoning attempts to gauge the level of involvement for the students in doing mathematics and contributing to meaning-making and reasoning in the class. This code is related to several of the other codes and seems to just corroborate what other codes have shown. Lesson A5 scored an overall 1.00, which is the lowest possible score, for this category (see Table 3). The highest overall score was given for lesson A3, which was the more hands-on lesson about area and perimeter. The overall score for lesson A3 was 1.50.

New segment codes. The MQI (2013) code of Students Work with Contextualized Problems attempts to indicate whether problems worked on in class are contextualized, i.e. real-world applications or statistics, or story problems. This code is also difficult to discern when students are given classwork out of the book. Only three of the eight classes had evidence of contextualized problems. However, the books used for classwork that I had the opportunity to review showed that there is a mix of contextualized problems and more abstract problems. So, although, there are no apparent contextualized problems during class time, that doesn't mean that the students did not work on contextualized problems in class (see Table 3).

Students Communicate about the Mathematics of the Segment looks at the frequency of student mathematical thinking that is contributed in a segment. This code however only applies when contributions are given in groups or the whole-class. Since the communication has to be at the whole-class or group level, the overall average would seem to indicate that there is less communication than is happening in the class, since segments when the students are working on classwork would be scored as not present. In this code an overall score of 0.00 would mean that the students are not communicating or their communication is trivial or incidental. None of the classes had so low a score, though A4, which was remediation of a previous lesson, had a very low score of 0.33, since much of the time was spent in working on classwork (see Table 3). C1, which had a score of 1.63, had one segment that scored a high for student communication (see Table C7). For C1, six of the eight segments scored had at least a mid. The remaining two segments scored a not present; although communication between teacher and students was occurring, it was not at the class or group level.

The MQI (2013) code of Mathematical Meaning and Sense-Making indicates whether the teacher and students address ideas like the meaning of and relationship between numbers, understanding the context associated with the numbers, whether the solution method and answer to a problem makes sense and making connections between mathematical ideas and representations, etc. The overall low score for this code would be a 0.00, which means that making sense of the mathematics as defined was not noticeable. Since the scores listed are overall then the average would be overall all segments. The highest score in this code was 1.40, for A1 (see Table 3). The teacher spent a good portion of time during the lesson explaining pi and how it relates to the diameter, radius, and circumference. The lowest score for this was 0.33 for class A4, which was remediation of previously covered material.

Whole lesson codes. The MQI (2013) also includes 5-point whole lesson codes which give an overview of different aspects of the classroom and classroom instruction. Each whole lesson code has a statement for which the coder decides how true the statement is for a lesson on a scale of 1 to 5. A score of 1 would mean that the statement is not true for the lesson. The names of the codes reflect the statements (see Table 4). The default score is 3, meaning the statement is somewhat true for the lesson, and could be considered comparable to an average lesson in the U.S. A 5 would mean the statement is very true of the lesson.

Reviewing the scores reveals some aspects of the lessons that stand out. For instance, C1 was a strong lesson scoring above average on almost all of whole lesson code categories (see Table 5). Looking at the segment codes and whole lesson codes, this lesson had positive elements in many of the categories. Scores of 3 would indicate that the lesson is at the level of what might be considered an average U.S. lesson. This is the default score in the MQI for the whole lesson codes. A score above 3 would be a lesson above average. Twelve of the seventy-two scores are 4.0 or higher showing that both raters felt the lesson was above average in those categories.

Table 4

Whole Lesson Code Statements

Whole Lesson Code	Code Statement
Teacher Uses Student Ideas	Teacher uses student ideas and solutions to move the lesson forward.
Teacher Attends to and Remediates Student Difficulty	Teacher attends to student difficulty with the material.
Students are Engaged	Classroom environment is characterized by engagement.
Classroom is Characterized by Mathematical Inquiry	Students participate in the mathematics of the lesson in a substantive way – by asking questions, volunteering their ideas, confirming their understanding of the material, piping up about a different solution method, etc.
Lesson Time is Used Efficiently	Lesson time is used efficiently; class is on task, and behavior issues do not disrupt the flow of the class.
Density of the Mathematics is High	"Density" of mathematics is high, in the sense that the class is working through many problems/tasks/concepts and the pace is reasonable or high.
Launch of Task	Launch of the mathematical task(s) was mathematically sensible, well-designed, clear and not confusing to children. Can students start working on a task(s) once launched?
Mathematics of the lesson is Clear and not Distorted	Mathematics of the lesson is clear and not distorted.
Tasks and Activities Develop Mathematics	The tasks and activities done by the class contribute to the development of mathematical ideas, procedures, etc. In other words, does the lesson have directionality, in the sense that it is appropriately developing mathematical ideas, or is the lesson off-track?

From "Mathematical Quality of Instruction (MQI): NCTE – Year 3 by Learning Mathematics for Teaching and H. Hill, 2013. Copyright [2013] by Learning Mathematics for Teaching and Heather Hill. Reprinted with permission.

The best scores overall for the Whole Lesson Codes are seen in the code of Mathematics of the Lesson is Clear and not Distorted with all but one of the lessons receiving 4.0 or higher and an average across the lessons of 4.44 (see Table 5). This fits to some degree the scores given in the category of Errors and Imprecisions (see Table 3). The next two highest overall average scores were Teacher Attends to and Remediates Student Difficulty and Lesson Time is Used Efficiently. In all the classes remediation, which included both procedural and conceptual remediation, did occur giving at least a 3 for all the classes. Lesson Time is Used Efficiently had only one score below a 3 and was given because of the amount of time it took for students create shapes with a certain perimeter or area (see Table 5). Not much off-task behavior was seen in the videos analyzed and the transitions during class time while not perhaps the most efficient also did not take excessive time (see Table 4). And as seen with Classroom Work is Connected to Mathematics, mathematics happened throughout class time (see Table 4, Table 2). Looking at the average of all the lessons, the code of Classroom is Characterized by Mathematical Inquiry has the lowest average across the codes. This is mostly because many of the questions covered in the lessons were largely procedural in nature. Half of the lessons, though, did score 3 or above on this category.

Table 5

Whole Lesson Code Averages

Whole Lesson Code	A1	A2	A3	A4	A5	A6	C1	C2	Avg
Teacher Uses Student Ideas	3.0	3.5	3.5	3.0	3.5	4.0	5.0	3.0	3.56
Teacher Attends to and Remediates Student Difficulty	4.0	3.5	3.5	3.5	3.5	4.0	3.5	3.5	3.63
Students are Engaged	3.5	3.5	3.5	3.5	3.5	3.5	4.0	3.5	3.56
Classroom is Characterized by Mathematical Inquiry	3.0	2.5	2.5	2.5	3.0	3.5	4.5	2.5	3.00
Lesson Time is Used Efficiently	3.5	4.0	2.5	3.5	4.5	3.5	4.5	3.0	3.63
Density of the Mathematics is High	3.0	3.5	3.0	3.0	4.0	4.0	4.0	3.5	3.50
Launch of Task	2.5	3.0	3.5	3.0	3.5	3.0	4.0	3.0	3.19
Mathematics of the Lesson is Clear and not Distorted	4.0	5.0	3.5	4.5	5.0	4.5	4.5	4.5	4.44
Tasks and Activities Develop Mathematics	3.5	3.5	3.0	3.5	3.0	4.0	3.0	3.0	3.31

Other whole lesson codes. Four other whole lesson codes are included in the MQI (2013). These codes are Teacher Uses Real World Examples, Students Conducts Error Analysis, Whole Lesson Mathematical Quality of Instruction, and Guess at Typical Mathematical Quality of Instruction. I will talk about these codes and the scores associated with them, except for Students Conduct Error Analysis, which was not found in any of the lessons.

The MQI (2013) whole lesson code of Teacher Uses Real World Examples indicates whether the teacher connects the math to the students' everyday life. This is different than the use of contextual problems in that the examples aren't worked on but are mentioned as situations in which the math would be used in the real world. In four of the eight lessons teachers gave at least one example of when the mathematics would be used in real life (see Appendix C). One real world example was that percentages could be used to compare the weights of people. Another real world example is when the teacher explained that the diameter is used to measure a hockey puck.

The Whole-Lesson Quality of Instruction (MQI) gives an indication of the coders' assessment of how the teaching quality compares with average instructional quality in the U.S. (see Table 6). The Guess at Typical Mathematical Quality of Instruction (MQI-T) gives an indication of whether the coder feels this lesson is typical for the teacher or whether their lessons might be typically better or worse. The coders are not to attempt to use averages of all the codes to decide these codes but instead are to make a judgment of their overall impression of the class instruction. Both coders gave only scores of 3 and 4 for all of the lessons. In fact, one rater gave only 3's for the MQI and MQI-T, except for lesson C1.

Table 6

Overall MQI (2013) Scores

Overall MQI Code	A1	A2	A3	A4	A5	A6	C1	C2	Avg
Whole-Lesson Mathematical Quality of Instruction (MQI)	3.0	3.5	3.0	3.0	3.5	3.5	4.0	3.0	3.31
Guess at Typical Mathematical Quality of Instruction (MQI-T)	3.5	3.5	3.0	3.5	3.5	3.5	4.0	3.5	3.50

Interviews

The interviews included five questions: Was this a typical lesson? How do you feel the lesson went? What went well in the lesson and why? What would you like to improve in the lesson and why? What is your process for preparing a lesson? The interviews were conducted the same day usually immediately after the class taught, though for two of the interviews there was a delay of an hour or two. Eight interviews were conducted of four teachers. The teachers recorded by the University of Helsinki were not interviewed. Six of the interviews were conducted in Finnish while two were conducted in English, by the teachers' choice. I will talk about some of the things that I learned from the interviews.

The first and second questions were, in some respects, interrelated. In response to the first question for 6 of the classes the teacher said that the lesson was normal, for one class the students were more cooperative and for the remaining class the students were quieter. The second question often was answered as a continuation of the first. In four of the interviews the teacher stated that the lesson went well though only one of the teachers explained further why the lesson went well. He stated that the lesson went well because his objective was met. One teacher explained that the lesson was normal even though he had thought that the camera in the classroom would change how the students responded. Another teacher mentioned that the announcements interrupted the students' attention. Other qualifications were that the students were more tired or that there were problems with the previous night's homework.

The third question asked what went well in the lesson. In four of the responses the teacher talked about the students participating well in the lesson. Two of the comments talked about how the students worked quietly. One of the comments was that nothing went exceptionally well though the students didn't have their cellphones out. In the researcher's opinion this class seemed to have the same level of participation as most of the other classes.

The fourth question dealt with how the teacher would like to improve the lesson. There weren't any similarities in the comments but the responses may reveal something about what the teachers consider in their lessons. In one class, the teacher would have liked to use more concrete tools and ideas. In another class the concern was the amount of time spent on cutting out different shapes to explore surface area and perimeter. Studying the material in more depth was the concern of one class. While another teacher was concerned that the students should be allowed to go through the curriculum quicker. One teacher expressed the concern that it would have been good if a real-world example had been used for the lesson instead. For another teacher, the homework was another concern. The final concern that was given was that the students were too loud while being a "very, active discussing group" of students.

Lesson preparation was the subject of the fifth question. More specific sub-questions would perhaps have helped in obtaining more information about the lesson planning process. Three of the four teachers made reference to the use of the textbook in preparing their lessons. Teacher 1 spent an hour thinking about the material and deciding on what main point he wanted the students to learn using the textbook material. Teacher 1 also spent time each lesson assessing what additional things to incorporate, including supports for students with special needs.

Teacher 2 would plan to review and incorporate previously learned knowledge, including knowledge from previous grades, so that he could build on that knowledge with the new material. Teacher 3 had lesson plans, some written down on paper and some just post-it notes, stuck in the teacher's manual. This teacher would also use on-line quizzes at the end of class to assess the understanding of the students to use in preparation for the next class. The last teacher interviewed used the text to select examples for use the next day, though wasn't afraid to make up new, related problems as well.

Observations

While in Finland I had the opportunity to not only record 8 classrooms but also to observe the teaching in 1st through 5th grade classrooms in peruskoulu, basic education. I am very grateful for the kindness and welcome that the teachers, students, and principals extended to us in all the schools and classrooms we visited, especially in the classrooms I also recorded.

In all the classrooms, I was impressed by the focus of the students in these classrooms and feel that their focus can be explained through several reasons. These reasons are the class breaks, the accommodation for movement, the relaxed feel between teachers and students, and the manner of teaching new material, especially in the elementary schools.

Finnish schools allow 15 minute breaks per 45 minutes of class time at all levels, from elementary to secondary, though some class times are longer at some schools. This allows the students time to move, relax, and even socialize and return to class better able to focus and learn. Recess time of greater than 15 minutes at a time has been shown to improve classroom behavior and educational activities (Barros, Silver, & Stein, 2009; Pelligrini & Smith, 1993; Rasberry, Lee, Robin, et al., 2011)

Also in the elementary school I observed the use of movement in the classroom which was also intended to aid in the focus of children. Underneath each desk in the lower elementary classes a rope was strung allowing students to move. Students would rest their feet on the rope and could move their feet and legs back and forth as they worked on material at their desks. Having substituted for many years at elementary and secondary schools, I was impressed with the level of focus of all the students in the classrooms.

In all the grades from elementary to secondary that I visited, students and teachers seemed more relaxed. Students seemed comfortable asking the teacher questions and volunteering answers. There are probably many reasons for this, possibly including having the same teacher for several grades, the caring attitude of the teachers, the use of first names for the teachers, the removal of shoes while in school, the lack of high stakes testing and/or the use of breaks and movement, etc. Whatever the reason, the feel in the classroom is more relaxed. This could be contributing to the success and education of Finnish students.

Math teaching at the elementary school level also appears to be context driven. Instead of rote memorization and procedure, students are taught math concepts using context. I observed a math lesson where the students were working on addition. The teacher called on students to help demonstrate the principle. Students were asked to come up to the front of the class as physical representations of the problem. For example, the teacher asked 1 girl to come up to the front, then 2 boys, followed by 3 girls. The teacher counted each group then asked how many total students were at the front of the classroom. Several examples of this nature were gone over by the class. If a student volunteered an incorrect answer the teacher took time to talk about the student's response and how to arrive at the correct total. Students then worked on additional problems using manipulatives, like macaroni or blocks or other items. In the elementary classes

I observed real-world context, like with counting students or with how much sale items cost, etc., being used to introduce and illustrate new material.

At the secondary level I saw a combination of both real-world situations and more abstract instruction. The secondary math books I had a chance to look at also used context in the introduction of the material. Homework includes real-world problem solving and more abstract work; problems of both types are given.

In the United States, there has been a lot of discussion about the health and nutrition of our children, especially in the context of school lunches. Even then, when I read how all Finnish students receive a free, hot lunch every day at school, I didn't realize the value of that lunch. In the United States I have seen how lunches have gone from being prepared on-site and served on real dishes to foods that are pre-packaged and prepared off-site then served on styrofoam in prescribed portions. The school lunch of my generation is not the school lunch of our children. When I had the privilege to eat in the cafeterias of the schools in Finland, I realized what school lunch could mean. The lunches were served buffet style with fresh foods and foods prepared on-site. Students had two options of entrée, several vegetables and/or fruits, and multiple choices of whole grain bread with real butter to choose from. They also had milk or milk alternative to drink. Students could dish up the types and amount of food they desired onto real plates with silverware. Students then cleaned up their own plates, glasses and eating utensils and put them in dishwasher trays. Obesity didn't seem to be an issue in any of the schools I visited, since I only saw one student who might be considered overweight. Imagine the impact on children's learning to have them eating healthy, nutritious food and to have them feeling full, not hungry. Perhaps this contributed to the relaxed feel in the classrooms and the focus of the students

The WILMA system, used by the Finnish school system, is a secure system that allows teachers and students to communicate. Teachers and parents e-mail each other through this secure system, which protects the student's private information. This aids the parents and teachers in helping their students succeed.

The use of classroom aides and special education in Finland is designed to meet the varied needs of the students. Classroom aides assisted the teachers in helping any students needing help during class time. Although I didn't have much opportunity to observe the use of special education, using special education to assist any student with needs as the needs occur is surely beneficial (Sahlberg, 2011). Imagine helping children when there is a need instead of several years later when students have a history of failure and misunderstanding.

Secondary school class size in Finland is less than the typical class size in many schools in the U.S. None of the classes that I visited had more than 25 students, even in the metropolitan area of Helsinki. In speaking to a principal of a secondary school, I was told that he considered 25 students to be the maximum number of students in a class and tried to keep class size between 15 and 20 students with 15 being the optimal number. Lower class size appears to be the norm in Finland. I think these things that I observed contribute to the relaxed feel in the classroom.

Research Questions

In this section I will talk about how my analysis answers my three research questions. My three questions are: What are the mathematical interactions between teachers and students in the Finnish mathematics classroom like? Are the mathematical interactions of high or low quality with respect to cognitive demand? Are there any patterns to the interactions that might help in better understanding Finnish mathematics instruction?

Mathematical interactions. Since in all the classes analyzed all the segments but two were coded as having as being connected to mathematics, regular mathematical interactions occurred between the teachers and students throughout the classes. The interactions were with the whole class or on an individual basis with the majority of the whole class interactions happening in the first half of class time as evidenced by the code Students Communicate about the Mathematics of the Segment (see Appendix C). Individual mathematical interactions between the teacher and the students were the most common during class work time as all of the teachers walked around the class remediating student difficulties on an individual basis during the latter part of the classes.

As the Imprecision in Language and Notation code shows (see Table 3), the language used by the teachers was clear and correct laying a solid foundation for communication and interactions between the teachers and students. From the Mathematical Language code we see that the teachers used the mathematical language and the students seemed to understand what was meant by the language (see Table 2). The notation used was also understandable and correct with only one small problem as mentioned under the Imprecision in Language and Notation code.

Teachers sought mathematical interactions with the students through several means. The teachers noticed student thinking by seeking examples of student work on the homework at the beginning of class (see Typical Lesson). They also asked questions of students during the lessons and responded to students' questions and answers as evidenced by the Teacher Uses Students Mathematical Contributions and the Teacher Uses Student Ideas codes (see Table 5). Although much of the comments and questions by the students were not very deep mathematically, they still occurred regularly and showed that teachers were interacting with the students about the mathematics and students were attempting to understand the material and interact with the teacher about the mathematics. The whole lesson code of Teacher Uses Student Ideas confirms that, for the most part, teachers are seeking student ideas to use during the lesson.

Cognitive demand. As stated previously, high cognitive demand tasks are tasks that are given to students that cause them to think, reason and make sense of the mathematics in the classroom. Instances of high cognitive tasks, as evidenced by the Explanations, Students Provide Explanations, Linking and Connections, and Multiple Procedures or Solution Methods codes, etc., were present throughout the classes as well as periods of Enacted Task Cognitive Activation (see Table 3). While there wasn't a consistent pattern of specific cognitively demanding activities, these activities were present in the form of explanations (as defined by the MQI), making connections, sense making of the mathematics, and classroom activities that were more cognitively demanding (see Table 2, Table 3). For the most part, few of the lessons could be considered outstanding with respect to cognitive demand but cognitively demanding tasks did occur at some point for all of the lessons, except A5, though even that class had some sense-making of the mathematics (see Mathematical Meaning and Sense-Making in Table C4).

Patterns in interactions. The patterns of interactions aligned fairly closely with the class time use, whether it was lesson time or classwork time (see Typical Lesson). During class time teachers would ask questions and students would respond as evidenced by the Teacher uses Student mathematical Contributions and the Students Communicate about the Mathematics of the Segment codes (see Table 2, Table 3). Much of the interactions during instruction time were IRE, though there were instances of more complex interactions IRE as evidenced by the Teacher Uses Student Mathematical Contributions code (see Table 2), for example, when a teacher and students sought to help a student who was struggling with understanding the relationship between the diameter and circumference of a circle at the end of class.

Interactions were fairly consistent throughout the classes, as mentioned previously. The teachers did not fail to respond to student comments and questions that were made during the lesson. Overall, students also seemed fairly willing to contribute throughout the lesson as determined by the code Students Communicate about the Mathematics of the Segment. The teachers were also willing to use those contributions beyond the basic IRE throughout the class as shown by the code Teacher Uses Student Mathematical Contributions.

Chapter 5: Discussion

When this research started, I was investigating certain aspects of the classroom that I felt were important in understanding why Finnish students were succeeding. I had hoped to find rich mathematical interactions characterized by explanations, connections to other mathematical knowledge, mathematical sense-making, and solid language throughout class time. I also hoped to discover good use of student thinking by the teacher throughout class time. These characteristics were present, though perhaps not to the degree I had envisioned. Other high cognitive demand activities, like linking and connections and mathematical sense-making were also present but not in great amounts. The mathematical language was solid but while students seemed to understand the mathematical language there weren't behaviors like pressing the students to use the language. The teachers did make efforts to notice student thinking throughout class time. From the beginning of class they showed the importance of noticing student thinking by reviewing student work on the homework. During instruction time, for the most part, the teachers asked questions to assess understanding and seek student input on the mathematics of the lesson. During class work time, the teachers moved among the students to remediate and evaluate the understanding of the students of the material. The interviews also suggested that student participation is important during the lesson since in four of the interviews the teacher commented that what made the lesson go well was the student participation in the lesson. Research suggests that when teachers notice student thinking and integrate it into instruction that student learning outcomes are improved (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996; Kazemi & Franke, 2004; Stein, Grover & Henningsen, 1996). Finnish teachers are noticing student thinking in their classrooms which most likely is contributing the success of their students.

Other factors of these classrooms are also of note. Some of these factors are the level of Errors and Imprecisions and the amount of class time spent on mathematics. Errors and Imprecisions in the classrooms studied were fairly low. The mathematics presented by the teachers was for the most part clear and error-free. This would seem to be a factor in helping students to learn and understand the mathematics presented while eliminating confusion on the subject matter.

In the classes I studied, math occurred almost the whole class time, partially because of the focus of the students. The transitions were unexceptionable without large amounts of wasted time. Stevenson and Stigler noticed a correlation between the classrooms of high performing countries and lower performing countries with respect to academic learning time. Stevenson and Stigler (1992) noted that in U.S. elementary classrooms only 65% of academic class time in fifth grade was spent on academic subjects, which had decreased from previous years. The opposite occurred in high performing countries, like Japan, where time spent the academic subjects increased during class time over the grades. Likewise, most of the time in the Finnish mathematics classrooms I studied was spent on mathematics. Students focused on the mathematics presented throughout the class which would allow students time to work with and opportunities to gain understanding of the mathematical content.

Hiebert and Grouws (2007) argue that for students to have opportunity to learn the tasks and activities need to be clear with clear goals, student engagement should be considered as well as students' prior knowledge. From my observations I think Finnish students are given these opportunities to learn for various reasons including the reasons listed. As mentioned before, teachers gave about three examples, sometimes less, on the mathematical topic to be covered. The objective of the lesson seemed very clear in the lessons given, like in Lesson Example 1, when the teacher stated they were going to learn how to calculate the surface area of a cylinder, explained how to find the surface area of a cylinder and then gave an example by finding the surface area of a cylinder. The lessons in all the classes seemed to have a clear objective and were direct and clean of errors without much loss of time on unrelated matters.

Since teachers and students often spend three years together in the mathematics classroom, the teachers are also aware of the students' previous mathematical experience and knowledge. As noted in the interview section by one of the teachers interviewed, this knowledge of student understanding and capability is used to help plan for upcoming lessons. Use of students' previous knowledge in the lesson and lesson planning would help to increase students' opportunities to learn.

I also noted previously and in my observations that students were focused in class. This would promote opportunities to learn (Hiebert & Grouws, 2007). Many reasons could be given for this focus, for instance, the 15 min. breaks, the relationships between teacher and students, addressing of student difficulties through remediation and additional help, the nutritious, filling lunches, parental support, and the small class sizes, etc. Hiebert and Grouws (2007) mention that the attentiveness of students is a mediating factor in students' opportunity to learn. Finnish schools seem to be affecting this mediating factor in a positive way promoting students' opportunity to learn.

The evolution of the Finnish school system has been a process of decades. This evokes the thought expressed by Stigler and Hiebert (2004) "Teaching can only change the way cultures change: gradually, steadily over time as small changes are made in the daily and weekly routines of teaching". Stigler and Hiebert also say that teachers need more time to plan and reflect in order to make changes to their teaching and improve learning opportunities. Finland has less instructional time, allowing more planning and reflection time for their teachers (Sahlberg, 2011). This extra planning and reflection should be mentioned as another factor in the success of Finnish students.

Hiebert & Grouws (2007) stated "systems of teaching, not single features of teaching, affect students' learning". Finland, through persistent effort, is creating a system of teaching that addresses both internal and external factors that promote opportunities to learn for their students. They are making efforts to meet the needs of their students both inside and outside the classroom which allows students to focus in the classroom and learn mathematics.

Although Finland has a system of many interacting features, I think there are factors that could be explored that might yield benefits for students in the U.S. Research is suggesting that when teachers notice student thinking and integrate it into instruction that student learning outcomes are improved (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996; Kazemi & Franke, 2004; Stein, Grover & Henningsen, 1996). Improving noticing of student thinking in U.S. classrooms is a factor I feel would benefit our students.

Another factor that I feel should be explored is the 15 minute class breaks. The level of focus of the students, which I feel is, at least in part, due to the 15 minute class breaks, makes me question the elimination of recess, etc., to "maximize learning time" in the U.S. Perhaps this instead reduces effective learning time. Another researcher commented that in Japan the level of focus is high and they also have the longer classroom breaks. Could something as simple as 15 minute class breaks improve the focus and learning of our students at all levels?

This was a small study with only 8 classes analyzed from two regions of Finland, because of this strong conclusions on the nature of Finnish mathematics classrooms at the secondary level can not be made. This study though does give a look into what may be happening in these classrooms and suggests some of the factors that may be contributing to the success of Finnish schools. A larger study of classrooms throughout Finland might reveal more significant differences in the interactions that could add to our understanding of why Finnish schools are succeeding. An emphasis on conceptual learning in the elementary grades could be a major factor as well in their success. How much of an impact is a question that could be explored.

The Finnish school system seeks not only to address the internal factors that affect student learning but the external factors as well from parent involvement to sufficient, nutritious food and breaks to help with focus. Because of similarities to U.S. traditional instruction one might describe Finnish instruction as traditional instruction in a nurturing environment. How does this multi-faceted approach to student learning impact the learning of students?

This study used the MQI which has not been used to study secondary classrooms for the most part. Further research could also be done on how the MQI works with U.S. secondary classrooms and how it compares with Finnish classrooms.

Conclusion

The results of this study add to the claim that Finland is succeeding on international assessments, not for one or two specific, outstanding elements, but because of the overall interaction of many factors inside and outside of the classroom. The teachers I observed gave solid instruction, which was mostly error free. Students were focused and interacted with the teacher and other students throughout class time and seemed to be comfortable making comments and asking questions. Teachers and support staff, like special education teachers and aides, are available to help with student difficulties. Math happened throughout the entire class, without large breaks in learning, and students for the most part were focused throughout class time. Students also seem to be building their mathematical knowledge on a solid conceptual base begun in elementary school. Additional outside needs are also addressed, for example, sufficient breaks, healthy nutrition, and parent support. Multiple factors are being addressed in the Finnish school system which would contribute to the success of their students and they are able to do this with less instructional time and less stress caused by measures like high-stakes testing.

References

Ball, D. L., & Hill, H. C. (2008). Measuring teacher quality in practice. In D. H. Gitomer (Ed.), *Measurement issues and assessment for teaching quality*. Thousand Oaks, CA: Sage Publications.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*, 389-407.

Cobb, P., Wood, T., & Yackel, E. (1993). Discourse, mathematical thinking, and classroom practice. In E. A. Forman, N. Minick & C. A. Stone (Eds.), *Contexts for learning: Sociocultural dynamics in children's development* (pp. 91-119). New York: Oxford University Press.

Cohen, D. K. (2011). *Teaching and its predicaments*. Cambridge, England: Harvard University Press.

Cohen, D. K., Raudenbush, S. W., & Ball, D. L. (2003). Resources, Instruction, and Research. *Educational Evaluation and Policy Analysis, 25*(2), 119-142. doi: 10.3102/01623737025002119

Cohen, D. K., & Ball, D. L. (1999). Instruction, Capacity, and Improvement *CPRE Research Report Series, RR-43*. University of Pennsylvania: Consortium for Policy Research in Education.

Corey, D. L., Peterson, B. E., Lewis, B. M., & Bukarau, J. (2010). Are there any places that students use their heads? Principles of high-quality Japanese mathematics instruction. *Journal for Research in Mathematics Education, 41*(5), 438-478.

Fan, L., & Zhu, Y. (2007). Representation of problem-solving procedures: A comparative look at China, Singapore, and US mathematics textbooks. *Educational Studies in Mathematics, 66*, 61-75. doi: 10.1007/s10649-006-9069-6

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education, 27(4),* 403-434.

Goldhaber, D. D., & Brewer, D. J. (1997). Evaluating the effect of teacher degree level on educational performance. In W. J. Fowler (Ed.), *Developments in school finance, 1996* (pp. 197-210). Washington D.C.: National Center for Educational Statistics.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., Olivier, A., Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding*. Portsmouth, NH: Heinemann.

Hiebert, J. (2013). The constantly underestimated challenge of improving mathematics teaching. In K. R. Leatham (Ed.), *Vital directions for mathematics education research* (pp. 45-56). New York, NY: Springer.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning, 1* (Vol. 371-404). Charlotte, N.C.: National Council of Teachers of Mathematics and Information Age Publishing.

Hiebert, J., & Stigler, J.W. (2004). A world of difference: Classrooms abroad provide lesson in teaching math and science. *JSD, 25(4),* 10-15.

Hiebert, J., Stigler, J. W., Jacobs, J. K., Givvin, K. B., Garnier, H., Smith, M., . . .

Gallimore, R. (2005). Mathematics teaching in the United States today (and tomorrow): Results from the TIMMS 1999 Video Study. *Educational Evaluation and Policy Analysis*, 27, 111-132.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction, 26*(4), 430-511. doi: 10.1080/07370000802177235

Hill, H. C., & Learning Mathematics for Teaching. (2013). *Mathematical Quality of Instruction (MQI). Coding Tool*. University of Michigan: Learning Mathematics for Teaching.

Jacobs, J. K., & Morita, E. (2002). Japanese and American teachers' evaluations of videotaped mathematics lessons. *Journal for Research in Mathematics Education*, 33, 154-175.

Kazemi, E., & Franke, M. L. (2004). Teacher learning in mathematics: Using student work to promote collective inquiry. *Journal of Mathematics Teacher Education, 7*, 203-235.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classroom. *The Elementary School Journal, 102*, 59-80.

Kumpulainen, K., & Lankinen, T. (2012). Striving for educational equity and excellence: Evaluation and assessment in Finnish basic education. In H. Niemi, A. Toom & A. Kallioniemi (Eds.), *Miracle of education: The principles and practices of teaching and learning in Finnish schools* (pp. 69-81). Rotterdam, The Netherlands: Sense Publishers.

Kupiainen, S., Hautamaki, J., & Karjalainen, T. (2009). The Finnish Education System and PISA. *Ministry of Education Publications*.

Learning Mathematics for Teaching Project. (2011). Measuring the mathematical quality of instruction. *Journal of Mathematics Teacher Education*. doi: 10.1007/s10857-010-9140-1

Lampiselkä, J., Ahtee, M., Pehkonen, E., Meri, M., & Eloranta, V. (2007). Mathematics and science in Finnish comprehensive school. In E. Pehkonen, M. Ahtee & J. Lavonen (Eds.), *How Finns learn mathematics and science* (pp. 35-48). Rotterdam, The Netherlands: Sense Publishers

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.

Niemi, H. (2012). The societal factors contributing to education and schooling in Finland. In H. Niemi, A. Toom & A. Kallioniemi (Eds.), *Miracle of education: Teaching and learning in Finnish schools* (pp. 19-38). Rotterdam, The Netherlands: Sense Publishers.

OECD (2003). The PISA 2003 assessment framework – Mathematics, reading, science and problem solving knowledge and skills. Paris: OECD. http://www.oecd.org/pisa/pisaproducts/pisa2003/learningfortomorrowsworldfirstresultsfrompisa2003.htm

OECD (2006). PISA Country Profiles. http://pisacountry.acer.edu.au/

Pehkonen, E., Hannula, M. S., & Bjorkqvist, O. (2007). Problem solving as a teaching method in mathematics education. In E. Pehkonen, M. Ahtee & J. Lavonen (Eds.), *How Finns learn mathematics and science* (pp. 119-131). Rotterdam, The Netherlands: Sense Publishers.

Pehkonen, E., & Rossi, M. (2007). Some alternative teaching methods in mathematics. In E. Pehkonen, M. Ahtee & J. Lavonen (Eds.). *How Finns learn mathematics and science*: Sense Publishers.

Pirie, S. E. B. (1998). Crossing the gulf between thought and symbol: Language as (slippery) stepping-stones. In H. Steinbring, M. G. B. Bussi & A. Sierpinska (Eds.), *Language and communication in the mathematics classroom* (pp. 7 - 29). Reston, VA: National Council of Teachers of Mathematics.

Reinikainen, P. (2012). Amazing PISA results in Finnish comprehensive schools. In H. Niemi, A. Toom & A. Kallioniemi (Eds.), *Miracle of education: The principles and practices of teaching and learning in Finnish schools* (pp. 3 - 18). Rotterdam, Netherlands: Sense Publishers.

Sahlberg, P. (2011). *Finnish lessons: What can the world learn from educational change in Finland?* New York, NY: Teachers College Press.

Schleicher, Andreas (2007). Losing Our Edge, video broadcast, available from http://www.youtube.com

Seaberg, R. L. (2015). Mathematics lessons from Finland and Sweden. *Mathematics Teacher, 108*, 593-598.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*, 4-14.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33, 455-488*.

Stevenson, H. W., & Stigler, J. W. (1992). *The learning gap*. New York, NY: Simon & Schuster Paperbacks.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap*. New York, NY: The Free Press.

Tirri, K. (2012). The core of school pedagogy: Finnish teachers' views on the educational purposefulness of their teaching. In H. Niemi, A. Toom & A. Kallioniemi (Eds.), *Miracle of education: The principles and practices of teaching and learning in Finnish schools* (pp. 55-66). Rotterdam, The Netherlands: Sense Publishers.

Toom, A., & Husu, J. (2012). Finnish teachers as 'Makers of the Many': Balancing between broad pedagogical freedom and responsibility. In H. Niemi, A. Toom & A. Kallioniemi (Eds.), *Miracle of education: The principles and practices of teaching and learning in Finnish schools* (pp. 19-38). Rotterdam, The Netherlands: Sense Publishers.

Appendix A

Mathematical Quality of Instruction (MQI) **Linking and Connections**		
This code refers to teachers' and students' explicit linking and connections: Among different representations of mathematical ideas or procedures (e.g., a linear graph and a table both capturing a linear relationship) Among different mathematical ideas (e.g., proportionality and linearity; fractions and ratios, etc.) Across representations and mathematical ideas/procedures (e.g., discussing how linearity is captured in any of the following: a graph, a table, or a mathematical equation) Note: If links are made but underlying representation/idea is incorrect, do NOT count as linking and connections.		
Low	**Mid**	**High**
No linking and connections occur. Also score as low when connections are **completely** pro forma (e.g., "Yesterday we did adding fractions with like denominators, today we will do subtracting fractions with like denominators.").	Links and connections are present, but do not have the features included in high, or feature these only momentarily (e.g., "You can compare ratios the same way you compare fractions" or "You can see each step in the computation in this array model here.").	Links and connections are present with sustained, careful work characterized by one of the following features: Explicitness about how two or more ideas, procedures, or representations are related (e.g., pointing to specific areas of correspondence) OR Detail and elaboration about how two mathematical ideas, procedures, or representations are related to one another (e.g., providing information about under what conditions the relationship occurs; noting meta-features; discussing implications of relationship)

From "Mathematical Quality of Instruction (MQI): NCTE – Year 3 by Learning Mathematics for Teaching and H. Hill, 2013, p. 5. Copyright [2013] by Learning Mathematics for Teaching and Heather Hill. Reprinted with permission.

Appendix B

Table B1

Video Selection

Video	Grade	Teacher	Selected	Reasoning	Lesson Topic
A1	8	A	Yes	8th grade class, video fairly good quality	Circumference of a circle
A2	7	A	Yes	Only 7th grade class, video quality good	Exponential power rules
A3	6	B	Yes	Only 6th grade class, video quality good	Area and perimeter of 2D shapes
A4	9	C	No	Same teacher and topic as A6 but with less dialogue resulting in less usable data.	Surface area and volume of Prisms
A5	8	C	Yes	Good sound, dialogue	Percents
A6	9	C	Yes	Good sound, dialogue	Surface area and volume of Prisms and Cylinders
A7	8	D	No	Same teacher as A8, difficult to hear dialogue in a number of places	Percents
A8	8	D	Yes	Same teacher as in A7, same grade, can hear dialogue better	Percents
C1	9	H	Yes	All right sound, teacher clear, harder to hear the students	Research problem, polynomial multiplication
C2	9	F	Yes	Different than other lessons, good sound overall though there are a few sound issues, can't see students	Research problem, review of answers on test and possible extension problems - unsure
C3	9	E	No, time consider-ations*	All right sound, sometimes hard to hear students	Research problem, the equation of a line
C4	9	G	No, time consider-ations	Good sound, teacher clear, sometimes hard to hear the students, lesson format similar to the ones I recorded	Research problem, primes & prime factorization
C5	9	E	No	Poor recording quality made transcription and translation very problematic	Research problem, worksheet on something else

* Lack of time and resources to do translation

Appendix C

Table C1

Lesson A1 Segment Codes

MQI (2013) Segment Codes	S1	S2	S3	S4	S5
Classroom Work is Connected to Mathematics	Yes	Yes	Yes	Yes	Yes
Whole-Class Discussion	None	None	None	None	none
Richness of Mathematics					
Linking and Connections	Mid	Mid	Mid	Mid	Low
Explanations	Low	Mid	Low	Low	Low
Multiple Procedures or Solution Methods	Low	Low	Low	Low	Low
Developing Mathematical Generalizations	Low	Low	Low	Low	Low
Mathematical Language	Mid	Mid	Mid	Mid	Mid
Overall Richness of the Mathematics (4 pt. Scale)	Mid	Mid	Mid	Low/mid	Low
Working with Students and Mathematics					
Remediation of Student Errors and Difficulties	Low	Low	Low	Mid	Mid
Teacher uses Student Mathematical Contributions	Low	Low	Low	Mid	Mid
Overall Working with Students and Mathematics (4 pt.)	Low	Low	Low	Low/ Mid	Mid
Errors and Imprecisions					
Major Mathematical Errors	Low	Mid	Low	Low	Low
Imprecision in Language or Notation	Low	Low	Low	Low	Low
Lack of Clarity in Presentation of Mathematical Content	Low	Low	Low	Low	Low
Overall Errors and Imprecision (4 pt.)	Low	Low/mid	Low	Low	Low
Student Participation in Meaning-Making and Reasoning					
Students Provide Explanation	Low	Mid	Low	Low	Low
Student Mathematical Questioning and Reasoning	Low	Low	Low	Mid	Low
Enacted Task Cognitive Activation	Low	Low	Low	Low	Mid
Overall Student Participation in Meaning-Making and Reasoning	Low	Low/mid	Low	Low	Low/mid
Other Segment Codes (4 pt.)					
Students Work with Contextualized Problems	N.P. *	Low	N.P.	Low	Low
Students Communicate about the Mathematics of the Segment	Low	Low	Low	Low	N.P.
Small Group/Pair Discussion	N.P.	N.P.	N.P.	N.P.	Low
Mathematical Meaning and Sense-Making	Low	Mid	N.P.	N.P.	mid

* N.P. means Not Present

Table C2

Lesson A2 Segment Codes

MQI (2013) Segment Codes	S1	S2	S3	S4	S5	S6
Classroom Work is Connected to Mathematics	Yes	Yes	Yes	Yes	Yes	Yes
Whole-Class Discussion	None	None	None	None	None	None
Richness of Mathematics						
Linking and Connections	Low	Low	Low	Low	Low	Low
Explanations	Low	Mid	Mid	Mid	Low	Low
Multiple Procedures or Solution Methods	Low	Low	Mid	Low	Low	Low
Developing Mathematical Generalizations	Low	Low	Low	Low	Low	Low
Mathematical Language	Mid	Mid	Mid	Mid	Mid	Mid
Overall Richness of the Mathematics (4 pt.)	Low	Low/ Mid	Mid	Low/ Mid	Low	Low
Working with Students and Mathematics						
Remediation of Student Errors and Difficulties	Low	Low	Low	Mid	Mid	Mid
Teacher uses Student Mathematical Contributions	Mid	Mid	Mid	Low	Low	Low
Overall Working with Students and Mathematics (4 pt.)	Mid	Mid	Mid	Mid	Low/ mid	Low/ mid
Errors and Imprecisions						
Major Mathematical Errors	Low	Low	Low	Low	Low	Low
Imprecision in Language or Notation	Low	Low	Low	Low	Low	Low
Lack of Clarity in Presentation of Mathematical Content	Low	Low	Low	Low	Low	Low
Overall Errors and Imprecision (4 pt.)	Low	Low	Low	Low	Low	Low
Student Participation in Meaning-Making and Reasoning						
Students Provide Explanation	Low	Mid	Mid	Low	Low	Low
Student Mathematical Questioning and Reasoning	Low	Low	Mid	Low	Low	Low
Enacted Task Cognitive Activation	Low	Low	Mid	Low	Low	Low
Overall Student Participation in Meaning-Making and Reasoning	Low	Low/ mid	Mid	Low	Low	Low
Other Segment Codes (4 pt.)						
Students Work with Contextualized Problems	N.P.*	N.P.	N.P.	N.P.	N.P.	N.P.
Students Communicate about the Mathematics of the Segment	Mid	Mid	Mid	Low	N.P.	Low
Small Group/Pair Discussion	N.P.	N.P.	N.P.	N.P.	N.P.	N.P.
Mathematical Meaning and Sense-Making	N.P.	Low	Low	Low	Low	Low

* N.P. means Not Present

Table C3

Lesson A3 Segment Codes

MQI (2013) Segment Codes	S1	S2	S3	S4	S5	S6
Classroom Work is Connected to Mathematics	Yes	Yes	Yes	Yes	Yes	Yes
Whole-Class Discussion	Some	None	None	None	None	None
Richness of Mathematics						
Linking and Connections	Mid	Low	Low	Low	Mid	Low
Explanations	Low	Low	Low	Low	Low	Low
Multiple Procedures or Solution Methods	Low	Low	Low	Low	Low	Low
Developing Mathematical Generalizations	Low	Low	Low	Low	Low	Low
Mathematical Language	High	Mid	Mid	Mid	Mid	Mid
Overall Richness of the Mathematics (4 pt.)	Mid	Low	Low	Low	Mid	Low
Working with Students and Mathematics						
Remediation of Student Errors and Difficulties	Low	Mid	Mid	Low	Low	Low
Teacher uses Student Mathematical Contributions	Mid	Low	Low	Low	Low	Low
Overall Working with Students and Mathematics (4 pt.)	Mid	Low/ mid	Low/ mid	Low	Low	low
Errors and Imprecisions						
Major Mathematical Errors	Mid	Low	Low	Low	Low	Low
Imprecision in Language or Notation	Low	Low	Low	Low	Low	Low
Lack of Clarity in Presentation of Mathematical Content	Low	Low	Low	Low	Low	Low
Overall Errors and Imprecision (4 pt.)	Low/ mid	Low	Low	Low	Low	Low
Student Participation in Meaning-Making and Reasoning						
Students Provide Explanation	Low	Low	Low	Low	Low	Low
Student Mathematical Questioning and Reasoning	Mid	Low	Mid	Mid	Mid	Low
Enacted Task Cognitive Activation	Mid	Mid	Mid	Low	Low	Low
Overall Student Participation in Meaning-Making and Reasoning	Mid	Low/ mid	Low/ mid	Low/ mid	Low/ mid	Low
Other Segment Codes (4 pt.)						
Students Work with Contextualized Problems	Mid	N.P.	N.P.	N.P.	Low	Low
Students Communicate about the Mathematics of the Segment	Mid	Low	Low	Low	Low	Mid
Small Group/Pair Discussion	N.P.*	N.P.	N.P.	N.P.	N.P.	N.P.
Mathematical Meaning and Sense-Making	High	Low	Low	Low	Mid	N.P.

* N.P. means Not Present

Table C4

Lesson A5 Segment Codes

MQI (2013) Segment Codes	S1	S2	S3	S4	S5	S6
Classroom Work is Connected to Mathematics	Yes	Yes	Yes	Yes	Yes	Yes
Whole-Class Discussion	None	None	None	None	None	None
Richness of Mathematics						
Linking and Connections	Low	Low	Low	Low	Low	Low
Explanations	Low	Low	Low	Low	Low	Low
Multiple Procedures or Solution Methods	Low	Low	Low	Low	Low	Low
Developing Mathematical Generalizations	Low	Low	Low	Low	Low	Low
Mathematical Language	Mid	Mid	Mid	Mid	Mid	Mid
Overall Richness of the Mathematics (4 pt.)	Low	Low	Low	Low	Low	Low
Working with Students and Mathematics						
Remediation of Student Errors and Difficulties	Mid	Mid	Mid	Mid	Low	Low
Teacher uses Student Mathematical Contributions	Mid	N.P.	Low	Low	N.P.	N.P.
Overall Working with Students and Mathematics (4 pt.)	Mid	Mid	Mid	Mid	Low	Low
Errors and Imprecisions						
Major Mathematical Errors	Low	Low	Low	Low	Low	Low
Imprecision in Language or Notation	Low	Low	Low	Low	Low	Low
Lack of Clarity in Presentation of Mathematical Content	Low	Low	Low	Low	Low	Low
Overall Errors and Imprecision (4 pt.)	Low	Low	Low	Low	Low	Low
Student Participation in Meaning-Making and Reasoning						
Students Provide Explanation	Low	Low	Low	Low	Low	Low
Student Mathematical Questioning and Reasoning	Low	Low	Low	Low	Low	Low
Enacted Task Cognitive Activation	Low	Low	Low	Low	Low	Low
Overall Student Participation in Meaning-Making and Reasoning	Low	Low	Low	Low	Low	Low
Other Segment Codes (4 pt.)						
Students Work with Contextualized Problems	N.P.	N.P.	N.P.	Low	Low	Low
Students Communicate about the Mathematics of the Segment	Low	N.P.	N.P.	N.P.	N.P.	Low
Small Group/Pair Discussion	N.P.	N.P.	N.P.	N.P.	Low	N.P.
Mathematical Meaning and Sense-Making	Low	Low	N.P.	N.P.	N.P.	N.P.

Table C5

Lesson A6 Segment Codes

MQI (2013) Segment Codes	S1	S2	S3	S4	S5	S6
Classroom Work is Connected to Mathematics	Yes	Yes	Yes	Yes	Yes	Yes
Whole-Class Discussion	None	None	None	None	None	None
Richness of Mathematics						
Linking and Connections	Low	Mid	Mid	Low	Low	Low
Explanations	Low	Low	Mid	Low	Low	Low
Multiple Procedures or Solution Methods	Low	Low	Low	Low	Low	Low
Developing Mathematical Generalizations	Low	Low	Low	Low	Low	Low
Mathematical Language	Mid	Mid	Mid	Mid	Mid	Mid
Overall Richness of the Mathematics (4 pt.)	Low	Low/ mid	Mid	Low	Low	Low
Working with Students and Mathematics						
Remediation of Student Errors and Difficulties	Mid	Low	Low	Mid	Mid	Mid
Teacher uses Student Mathematical Contributions	Mid	Low	Mid	Mid	Low	Mid
Overall Working with Students and Mathematics (4 pt.)	Mid	Low	Mid	Mid	Mid	Mid
Errors and Imprecisions						
Major Mathematical Errors	Low	Low	Low	Low	Low	Low
Imprecision in Language or Notation	Low	Low	Low	Low	Low	Low
Lack of Clarity in Presentation of Mathematical Content	Low	Low	Low	Low	Low	Low
Overall Errors and Imprecision (4 pt.)	Low	Low	Low	Low	Low	Low
Student Participation in Meaning-Making and Reasoning						
Students Provide Explanation	Low	Low	Low	Low	Low	Low
Student Mathematical Questioning and Reasoning	Low	Mid	Mid	Low	Low	Mid
Enacted Task Cognitive Activation	Low	Low	Low	Mid	Low	Low
Overall Student Participation in Meaning-Making and Reasoning	Low	Low	Low	Low/ mid	Low	Low/ mid
Other Segment Codes (4 pt.)						
Students Work with Contextualized Problems	N.P.	N.P.	N.P.	N.P.	Pres.	Pres.
Students Communicate about the Mathematics of the Segment	Mid	Mid	Mid	Mid	N.P.	N.P.
Small Group/Pair Discussion	N.P.	N.P.	N.P.	N.P.	N.P.	Low
Mathematical Meaning and Sense-Making	Low	Low	Low	Low	N.P.	N.P.

Table C6

Lesson A8 Segment Codes

MQI (2013) Segment Codes	S1	S2	S3	S4	S5	S6
Classroom Work is Connected to Mathematics	Yes	Yes	Yes	Yes	Yes	Yes
Whole-Class Discussion	None	None	None	None	None	None
Richness of Mathematics						
Linking and Connections	Mid	Mid	Low	Low	Low	Low
Explanations	Low	Mid	Low	Mid	Low	Low
Multiple Procedures or Solution Methods	Low	Mid	Mid	Low	Low	Low
Developing Mathematical Generalizations	Low	Low	Low	Low	Low	Low
Mathematical Language	Mid	Mid	Mid	Mid	Mid	Low
Overall Richness of the Mathematics (4 pt.)	Mid	Mid	Mid	Mid	Low	Low
Working with Students and Mathematics						
Remediation of Student Errors and Difficulties	Low	Low	Mid	Mid	Mid	Mid
Teacher uses Student Mathematical Contributions	Low	Mid	Mid	Mid	Mid	Mid
Overall Working with Students and Mathematics (4 pt.)	Low	Low/ mid	Mid	Mid	Mid	Mid
Errors and Imprecisions						
Major Mathematical Errors	Low	Low	Low	Low	Low	Low
Imprecision in Language or Notation	Low	Low	Low	Low	Low	Low
Lack of Clarity in Presentation of Mathematical Content	Low	Low	Low	Low	Low	Low
Overall Errors and Imprecision (4 pt.)	Low	Low	Low	Low	Low	Low
Student Participation in Meaning-Making and Reasoning						
Students Provide Explanation	Low	Low	Low	Low	Low	Low
Student Mathematical Questioning and Reasoning	Low	Low	Low	Low	Mid	Mid
Enacted Task Cognitive Activation	Low	Low	Mid	Low	Low	Low
Overall Student Participation in Meaning-Making and Reasoning	Low	Low	Mid	Low	Low	Low
Other Segment Codes (4 pt.)						
Students Work with Contextualized Problems	N.P.	N.P.	N.P.	N.P.	N.P.	N.P.
Students Communicate about the Mathematics of the Segment	Low	Low	Low	Low	N.P.	N.P.
Small Group/Pair Discussion	N.P.	N.P.	N.P.	Low	Low	N.P.
Mathematical Meaning and Sense-Making	Low	Low	Low	N.P.	Mid	N.P.

Table C7

Lesson C1 Segment Codes

MQI (2013) Segment Codes	S1	S2	S3	S4	S5	S6	S7	S8
Classroom Work is Connected to Mathematics	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Whole-Class Discussion	S	S	N	N	N	N	N	N
Richness of Mathematics								
Linking and Connections	Low	Low	Low	Low	Low	Low	Low	Low
Explanations	Low	Low	Low	Low	Low	Mid	Low	Low
Multiple Procedures or Solution Methods	Low	Low	Low	Low	Low	Low	Low	Low
Developing Mathematical Generalizations	Low	Low	Low	Low	Low	Low	Low	Low
Mathematical Language	Mid	Mid	Mid	Mid	Mid	Mid	Mid	Mid
Overall Richness of the Mathematics (4 pt)	Low	Low	Low	Low	Low	Low/ mid	Low	low
Working with Students and Mathematics								
Remediation of Student Errors and Difficulties	Low	Low	Low	Low	Mid	Mid	Low	Low
Teacher uses Student Mathematical Contributions	Mid	Mid	Mid	Low	Mid	Mid	Low	Low
Overall Working with Students and Mathematics (4 pt.)	Mid	Mid	Low/ mid	Low	Mid	Mid	Low	Low
Errors and Imprecisions								
Major Mathematical Errors	Low	Low	Low	Low	Low	Low	Low	Low
Imprecision in Language or Notation	Low	Low	Low	Low	Mid	Low	Low	Low
Lack of Clarity in Presentation of Mathematical Content	Low	Low	Low	Low	Low	Low	Low	Low
Overall Errors and Imprecision	Low	Low	Low	Low	Low/ mid	Low	Low	Low
Student Participation in Meaning-Making and Reasoning								
Students Provide Explanation	Mid	Mid	Low	Low	Low	Low	Low	Low
Student Mathematical Questioning and Reasoning	High	Mid	Mid	Mid	Mid	Mid	Low	Low
Enacted Task Cognitive Activation	High	High	Mid	Low	Low	Low	Low	Low
Overall Student Participation in Meaning-Making and Reasoning (4 pt.)	Mid	Mid	Mid	Low	Low	Low	Low	Low
Other Segment Codes (4 pt.)								
Students Work with Contextualized Problems	N.P.	N.P.	N.P.	N.P.	N.P.	N.P.	N.P.	N.P.
Students Communicate about the Mathematics of the Segment	High	Mid	Mid	Mid	Mid	Mid	N.P.	N.P.
Small Group/Pair Discussion	N.P.	Low	Low	N.P.	N.P.	N.P.	High	Low
Mathematical Meaning and Sense-Making	Mid	Low	Low	Low	Low	Low	N.P.	N.P.

Table C8

Lesson C2 Segment Codes

MQI (2013) Segment Codes	S1	S2	S3	S4	S5	S6	S7
Classroom Work is Connected to Mathematics	Yes	Yes	Yes	No	Yes	yes	No
Whole-Class Discussion	None	None	None	None	None	None	None
Richness of Mathematics							
Linking and Connections	Low	Low	Low	Low	Low	Low	Low
Explanations	Low	Low	Low	Low	Low	Low	Low
Multiple Procedures or Solution Methods	Low	Low	Low	Low	Low	Low	Low
Developing Mathematical Generalizations	Low	Low	Low	Low	Low	Low	Low
Mathematical Language	Mid	Mid	Mid	Mid	Mid	Mid	Low
Overall Richness of the Mathematics (4 pt.)	Low	Low	Low	Low	Low	Low	Low
Working with Students and Mathematics							
Remediation of Student Errors and Difficulties	Mid	Low	Low	Low	Mid	Low	Low
Teacher uses Student Mathematical Contributions	Lo	Mid	Mid	Low	Low	Low	Low
Overall Working with Students and Mathematics (4 pt.)	Low/ mid	Mid	Low/ mid	Low	Low/ mid	Low	Low
Errors and Imprecisions							
Major Mathematical Errors	Low	Low	Low	Low	Low	Low	Low
Imprecision in Language or Notation	Low	Low	Low	Low	Low	Low	Low
Lack of Clarity in Presentation of Mathematical Content	Low	Low	Low	Low	Low	Low	Low
Overall Errors and Imprecision	Low	Low	Low	Low	Low	Low	Low
Student Participation in Meaning-Making and Reasoning							
Students Provide Explanation	Low	Low	Low	Low	Low	Low	Low
Student Mathematical Questioning and Reasoning	Low	Low	Low	Low	Mid	Low	Low
Enacted Task Cognitive Activation	Mid	Mid	Mid	Low	Low	Low	Low
Overall Student Participation in Meaning-Making and Reasoning	Low	Low/ mid	Low	Low	Low/ mid	Low	Low
Other Segment Codes (4 pt.)							
Students Work with Contextualized Problems	N.P.	N.P.	N.P.	N.P.	N.P.	N.P.	N.P.
Students Communicate about the Mathematics of the Segment	Low	Low	Mid	Low	Low	Low	N.P.
Small Group/Pair Discussion	N.P.	N.P.	N.P.	N.P.	N.P.	Low	N.P.
Mathematical Meaning and S-M	N.P.	Low	Mid	Low	Mid	Low	N.P.